UNIT

1

DESIGN OF CURVED BEAMS

1.1 CURVED BEAM

Curved beams are the parts of machine members found in C - clamps, crane hooks, frames of presses, riveters, punches, shears, boring machines, planers etc. In straight beams the neutral axis of the section coincides with its centroidal axis and the stress distribution in the beam is linear. But in the case of curved beams the neutral axis of the section is shifted towards the centre of curvature of the beam causing a non-linear [hyperbolic] distribution of stress. The neutral axis lies between the centroidal axis and the centre of curvature and will always be present within the curved beams.

1.2 STRESSES IN CURVED BEAM

Consider a curved beam subjected to bending moment M_b as shown in Fig. 1.1. The distribution of stress in curved flexural member is determined by using the following assumptions :

i) The material of the beam is perfectly homogeneous [i.e., same material throughout] and isotropic [i.e., equal elastic properties in all directions]

ii) The cross section has an axis of symmetry in a plane along the length of the beam.

iii) The material of the beam obeys Hooke's law.

iv) The transverse sections which are plane before bending remain plane after bending also.

v) Each layer of the beam is free to expand or contract, independent of the layer above or below it.

vi) The Young's modulus is same both in tension and compression.

In the Fig. 1.1 the lines 'ab' and 'cd' represent two such planes before bending. i.e., when there are no stresses induced. When a bending moment 'M_b' is applied to the beam the plane cd rotates with respect to 'ab' through an angle '$d\theta$' to the position 'fg' and the outer fibres are shortened while the inner fibres are elongated. The original length of a strip at a distance 'y' from the neutral axis is $(y + r_n)\theta$. It is shortened by the amount $yd\theta$ and the stress in this fibre is, $\sigma = E.e$ where σ = stress, e = strain and E = Young's Modulus

$$\text{i.e.,} \quad \sigma = -E\frac{yd\theta}{(y+r_n)\theta} \qquad \text{..... (i)}$$

Since the fibre is shortened, the stress induced in this fibre is compressive stress and hence negative sign.

The load on the strip having thickness dy and cross sectional area dA is 'dF'

$$\text{i.e., } dF = \sigma dA = -\frac{Eyd\theta}{(y+r_n)\theta}.dA$$

From the condition of equilibrium, the summation of forces over the whole cross-section is zero and the summation of the moments due to these forces is equal to the applied bending moment.

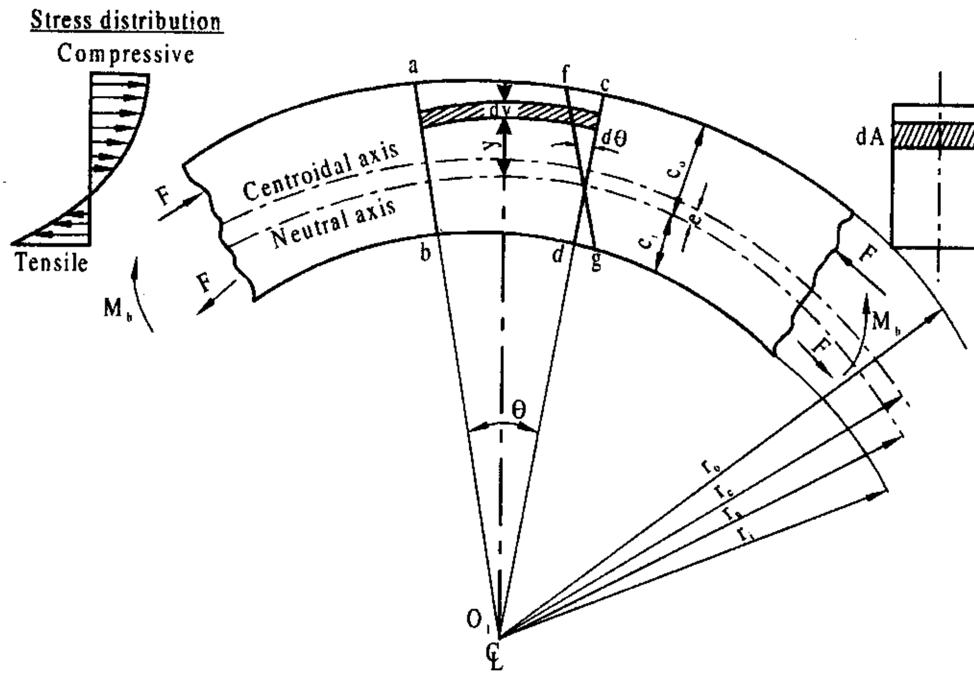

Fig. 1.1 [25.12 DDHB]

Let M_b = Applied Bending Moment

r_i = Inner radius of curved beam

r_o = Outer radius of curved beam

r_c = Radius of centroidal axis

r_n = Radius of neutral axis

℄ = Centre line of curvature

$$\text{i.e., } \int dF = 0$$

$$\therefore \frac{Ed\theta}{\theta}\int\frac{ydA}{(y+r_n)} = 0$$

As $\frac{Ed\theta}{\theta}$ is not equal to zero, $\therefore \int \frac{ydA}{(y+r_n)} = 0$ ..... (ii)

The neutral axis radius 'r_n' can be determined from the abvoe equation

If the moments are taken about the neutral axis,

$$M_b = -\int ydF$$

Substituting the value of dF, we get

$$M_b = \frac{Ed\theta}{\theta}\int \frac{y^2}{(y+r_n)} dA$$

$$= \frac{Ed\theta}{\theta}\int \left(y - \frac{yr_n}{y+r_n}\right) dA$$

$$= \frac{Ed\theta}{\theta}\int ydA \qquad \left[\because \int \frac{ydA}{y+r_n} = 0\right]$$

Since $\int ydA$ represents the statical moment of area, it may be replaced by A.e., the product of total area A and the distance 'e' from the centroidal axis to the neutral axis.

$$\therefore\ M_b = \frac{Ed\theta}{\theta} A.e \qquad \text{..... (iii)}$$

From equation (i) $\frac{Ed\theta}{\theta} = -\frac{\sigma(y+r_n)}{y}$

Substituting in equation (iii)

$$M_b = -\frac{\sigma(y+r_n)}{y} . A.e.$$

$$\therefore\ \sigma = -\frac{M_b y}{(y+r_n)Ae} \qquad \text{..... (iv)}$$

This is the general equation for the stress in a fibre at a distance 'y' from neutral axis.

At the outer fibre, $y = c_o$

$\therefore$ Bending stress at the outer fibre $\sigma_{b_o} = -\frac{M_b c_o}{Ae(r_n + c_o)}$

$$\text{i.e., } \sigma_{b_o} = -\frac{M_b c_o}{Aer_o} \qquad (\because r_n + c_o = r_o) \qquad \text{..... (v)}$$

where c_o = Distance from neutral axis to outer fibre. It is compressive stress and hence negative sign. At the inner fibre $y = -c_i$

$\therefore$ Bending stress at the inner fibre $\sigma_{b_i} = \dfrac{M_b c_i}{Ae(r_n - c_i)}$

i.e., $\sigma_{b_i} = \dfrac{M_b c_i}{Aer_i}$ $\quad (\because r_n - c_i = r_i)$ ..... (vi)

where c_i = Distance from neutral axis to inner fibre. It is tensile stress and hence positive sign.

Using usual notations prove that the moment of resistance M of a curved beam of initial radius R_1 when bent to a radius R_2 by uniform bending moment is $M = EAeR_1 \left(\dfrac{1}{R_2} - \dfrac{1}{R_1}\right)$

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Consider a curved beam of uniform cross section as shown in Figure 1.2. Its transverse section is symmetric with respect to the y axis and in its unstressed state, its upper and lower surfaces intersect the vertical xy plane along the arcs of circle AB and EF centered at O [Fig. 1.2a]. Now apply two equal and opposite couples M and M' as shown in Fig. 1.2 c. The length of neutral surface remains the same. θ and θ' are the central angles before and after applying the moment M. Since the length of neutral surface remains the same

$$R_1\theta = R_2\theta' \quad \text{..... (i)}$$

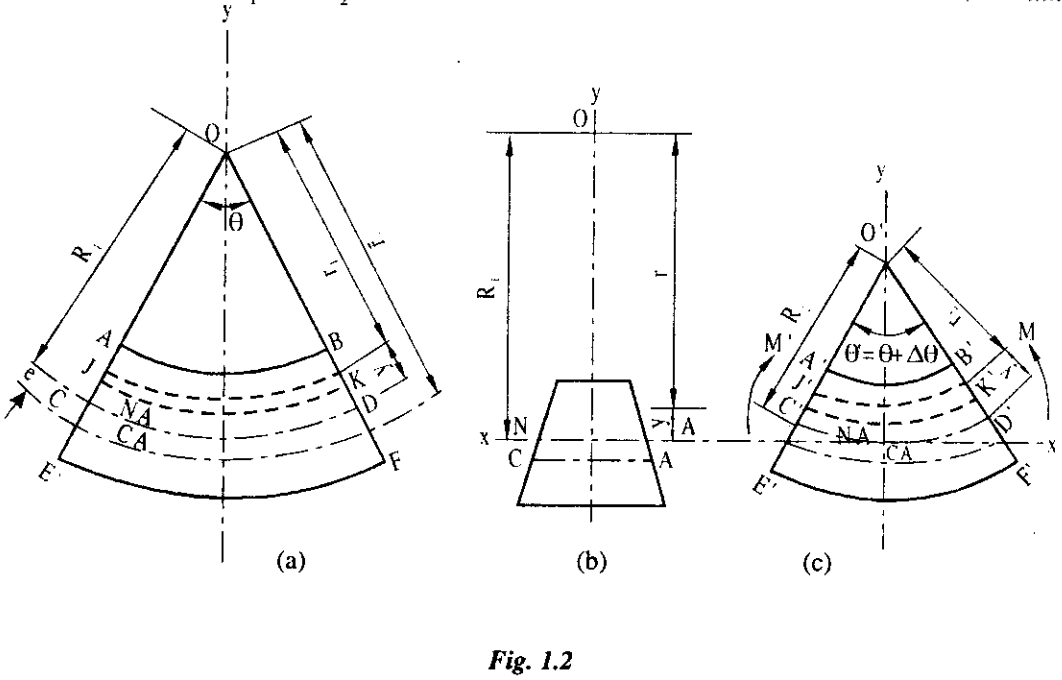


Fig. 1.2

Consider the arc of circle JK located at a distance y above the neutral surface. Let r_1 and r_2 be the radius of this arc before and after bending couples have been applied. Now, the deformation of JK, $\delta = r_2\theta' - r_1\theta$ ..... (ii)

From Fig. 1.2 a and c, $r_1 = R_1 - y$; $r_2 = R_2 - y$ ..... (iii)

$$\therefore \delta = (R_2 - y)\,\theta' - (R_1 - y)\,\theta$$

$$= R_2\theta' - \theta' y - R_1\theta + \theta y$$

$$= -y(\theta' - \theta) \quad [\because R_1\theta = R_2\theta' \text{ from equ (i)}]$$

$$\therefore \delta = -y\Delta\theta \qquad [\because \theta' - \theta = \theta + \Delta\theta - \theta = \Delta\theta] \qquad \text{..... (iv)}$$

The normal strain ϵ_x in the element of JK is obtained by dividing the deformation δ by the original length $r_1\theta$ of arc JK.

$$\therefore \epsilon_x = \frac{\delta}{r_1\theta} = -\frac{y\Delta\theta}{r_1\theta} = -\frac{y\Delta\theta}{(R_1 - y)\theta} \qquad \text{..... (v)}$$

The normal stress σ_x may be obtained from Hooke's law $\sigma_x = E\epsilon_x$

$$\therefore \sigma_x = -E\frac{\Delta\theta}{\theta}\left(\frac{y}{R_1 - y}\right) \qquad \text{..... (vi)}$$

$$\text{i.e. } \sigma_x = -E\frac{\Delta\theta}{\theta}\left(\frac{R_1 - r_1}{r_1}\right) \qquad (\because r_1 = R_1 - y) \qquad \text{..... (vii)}$$

Equation (vi) shows that the normal stress σ_x does not vary linearly with the distance y from the neutral surface. Plotting σ_x versus y, we obtain an arc of hyperbola as shown in Fig. 1.3.

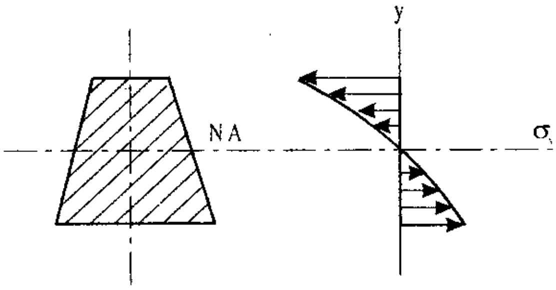


Fig. 1.3

From the condition of equilibrium the summation of forces over the entire area is zero and the summation of the moments due to these forces is equal to the applied bending moment.

$$\therefore \int \delta F = 0$$

$$\text{i.e., } \int \sigma_x dA = 0 \qquad \text{..... (viii)}$$

$$\text{and } \int(-y\sigma_x dA) = M \qquad \text{..... (ix)}$$

Substituting the value of the σ_x from equation (vii) into equation (viii)

$$-\frac{E\Delta\theta}{\theta}\int\left(\frac{R_1 - r_1}{r_1}\right)dA = 0$$

Since $\frac{E\Delta\theta}{\theta}$ is not equal to zero $\int\left(\frac{R_1 - r_1}{r_1}\right)dA = 0$

i.e. $R_1\int\frac{dA}{r_1} - \int dA = 0$

i.e., $R_1\int\frac{dA}{r_1} - A = 0$

$$\therefore R_1 = \frac{A}{\int\frac{dA}{r_1}}$$

$\therefore$ It follows the distance R_1 from the centre of curvature O to the neutral surface is obtained by the relation $R_1 = \frac{A}{\int\frac{dA}{r_1}}$ ..... (x)

The value of R_1 is not equal to the distance $\bar{r}_1$ from O to the centroid of the cross-section, since $\bar{r}_1$ is obtained by the relation,

$$\bar{r}_1 = \frac{1}{A}\int r_1 dA \qquad \text{..... (xi)}$$

Hence it is proved that in a curved member the neutral axis of a transverse section does not pass through the centroid of that section.

Now substitute the value of σ_x from equation (vii) into equations (ix)

$$\int\frac{E\Delta\theta}{\theta}\cdot\left(\frac{R_1 - r_1}{r_1}\right)ydA = M$$

i.e., $\frac{E\Delta\theta}{\theta}\int\frac{(R_1 - r_1)^2}{r_1}dA = M$ $\quad (\because r_1 = R_1 - y \text{ from iii})$

i.e., $\frac{E\Delta\theta}{\theta}\int\frac{\left(R_1^2 - 2R_1r_1 + r_1^2\right)}{r_1}dA = M$

i.e., $\frac{E\Delta\theta}{\theta}\left[R_1^2\int\frac{dA}{r_1} - 2R_1\int dA + \int r_1 dA\right] = M$

i.e., $\frac{E\Delta\theta}{\theta}\left[R_1^2\left(\frac{A}{R_1}\right) - 2R_1A + \bar{r}_1A\right] = M$ [using equations (x) and (xi)] i.e., $\frac{E\Delta\theta}{\theta}\left[R_1 A - 2R_1 A + \bar{r}_1 A\right] = M$

i.e., $\frac{E\Delta\theta}{\theta}\left[\bar{r}_1 A - R_1 A\right] = M$

$$\text{i.e., } E\frac{\Delta\theta}{\theta} = \frac{M}{A\left(\bar{r}_1 - R_1\right)} \qquad \text{..... (xii)}$$

$$\text{i.e., } \frac{E\Delta\theta}{\theta} = \frac{M}{Ae} \quad (\because e = \bar{r}_1 - R_1 \text{ from Fig. 1.2a}) \qquad \text{..... (xiii)}$$

Substituting $\frac{E\Delta\theta}{\theta}$ into equation (vi)

$$\sigma_x = -\frac{My}{Ae(R_1 - y)} \qquad \text{..... (xiv)}$$

$$\therefore \ \sigma_x = \frac{M(r_1 - R_1)}{Aer_1} \quad (\because r_1 = R_1 - y) \qquad \text{..... (xv)}$$

Equations (xiv) is the general expression for the normal stress σ_x in a curved beam.

To determine the change in curvature of the neutral surface caused by the bending moment M

From equation (i), $\frac{1}{R_2} = \frac{1}{R_1}\frac{\theta'}{\theta}$

$$= \frac{1}{R_1}\left(\frac{\theta + \Delta\theta}{\theta}\right)$$

$$= \frac{1}{R_1}\left(1 + \frac{\Delta\theta}{\theta}\right) = \frac{1}{R_1}\left[1 + \frac{M}{EAe}\right] \left\{\because \text{From equation (xiii)} \frac{E\Delta\theta}{\theta} = \frac{M}{Ae}\right\}$$

$$= \frac{1}{R_1} + \frac{M}{EAeR_1}$$

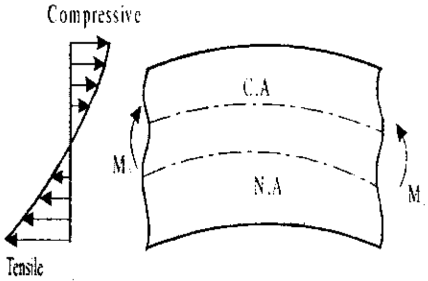
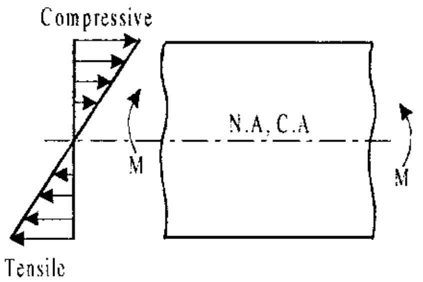
$$\text{i.e.,} \qquad \frac{1}{R_2} - \frac{1}{R_1} = \frac{M}{EAeR_1}$$

$$\therefore \ M = EAe\,R_1\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

Hence Proved.

List out the main differences between straight and curved beam. Also sketch the stress distribution pattern in a curved beam and compare it with a straight beam.

VTU, Feb. 2002, Jan/Feb 2003, Aug. 2001

Curved Beam	Straight Beam
1. Neural axis of the section does not coincide with its centroidal axis and it is shifted towards the centre of curvature of the beam	1. Neutral axis of the section coincide with its centroidal axis.
2. Stresses are not proportional to the distances of the fibres from the neutral axis and therefore it causes non-linear distribution of stress i.e., hyperbolic.	2. Stresses are proportional to the distance of the fibres from the neutral axis and hence the distribution of stress is linear.
3. The general expression for bending stress in a curved beam is, $\sigma_b = -\frac{M_b y}{Ae(R_i - y)}$ or $-\frac{M_b y}{Ae(y + r_n)}$	3. The general expression for bending stress in a straight beam is $\sigma_b = \frac{M}{I} \cdot y$
4. Stress distribution in curved beam is as shown in Fig. 1.4a  *Fig. 1.4 a*	4. Stress distribution in straight beam is as shown in Fig. 1.4b.  *Fig. 1.4b*

Symbols

℄ = Centre line of curvature

M_b = Bending moment at the centroid

CA = Centroidal axis

NA = Neutral axis

r_o = Radius of outer fibre or outer Fibre radius of curved beam

r_i = Radius of inner fibre or Inner Fibre radius of curved beam

r_c = Centroidal axis radius

r_n = Neutral axis radius

e = Eccentricity = Distance from centroidal axis to neutral axis

c_i = Distance from neutral axis to inner radius of curved beam

Symbols used in curved beam

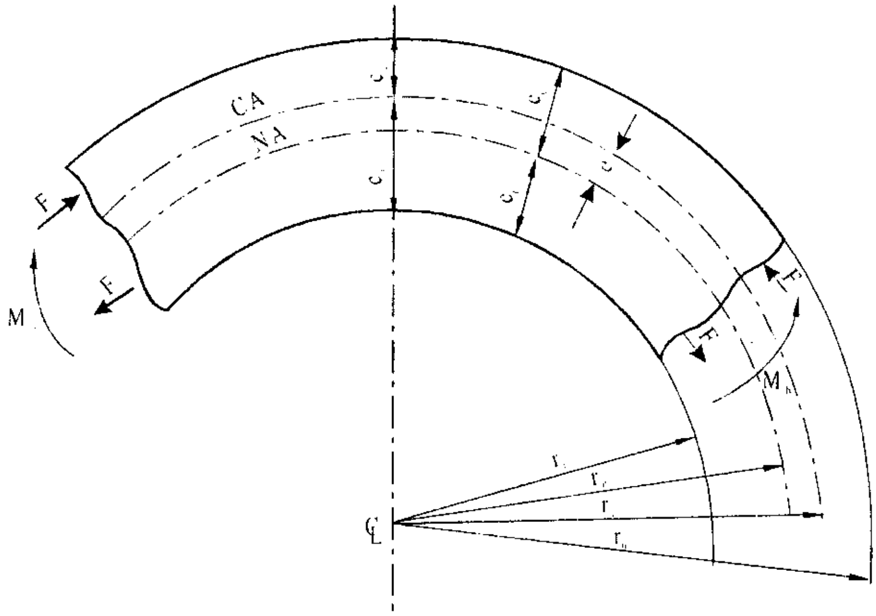


Fig. 1.5

c_o = Distance from neutral axis to outer radius of curved beam

c_i = Distance from centroidal axis to inner radius of curved beam

c_2 = Distance from centroidal axis to outer radius of curved beam

F = Applied load or Force

l = Distance from force to centroidal axis at critical section

A = Area of cross section

σ_d = Direct stress

σ_{bi} = Bending stress at the inner fibre

σ_{bo} = Bending stress at the outer fibre

σ_{ri} = Combined stress at the inner fibre

σ_{ro} = Combined stress at the outer fibre

τ_{max} = Maximum shear stress = 0.5 σ_{max}

Example : 1.1

Determine the maximum tensile, compressive and shear stress induced in a 'C' frame of a hydraulic portable riveter shown in Fig. 1.6 a

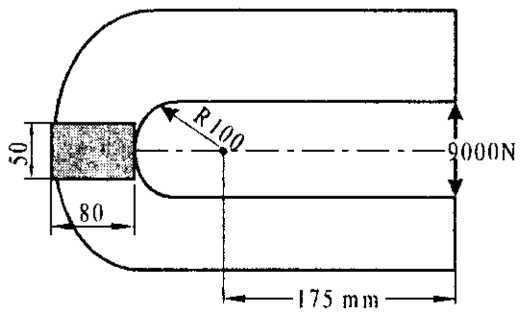


Fig. 1.6a

Solution :

Redraw the critical section as shown in Fig. 1.6b

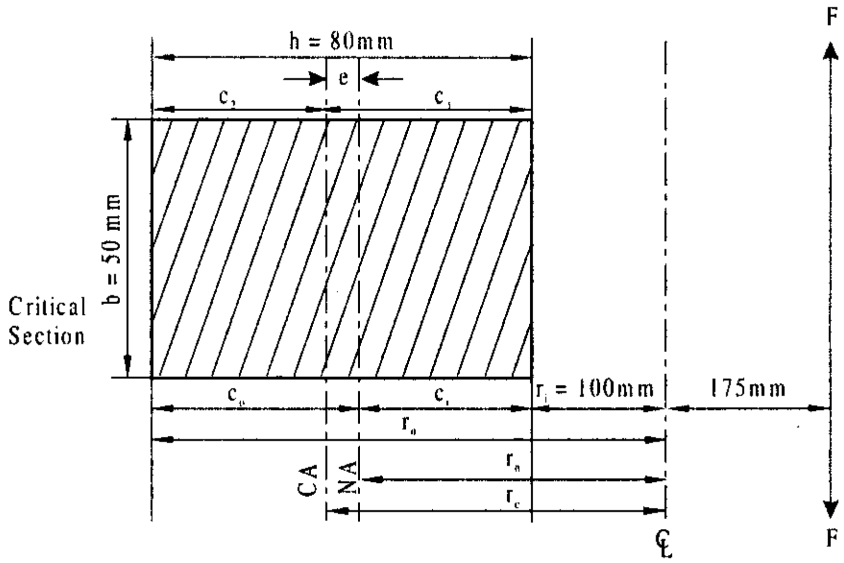


Fig. 1.6b

Inner radius of curved beam $r_i = 100 \text{mm}$

Outer radius of curved beam $r_o = 100+80 = 180 \text{mm}$

Radius of centroidal axis $r_c = 100 + \frac{80}{2} = 140 \text{mm}$

Radius of neutral axis $r_n = \frac{h}{ln\left(\frac{r_o}{r_i}\right)} = \frac{80}{ln\left(\frac{180}{100}\right)} = 136.1038 \text{ mm}$ 25.61

Distance of neutral axis to centroidal axis $e = r_c - r_n = 140 - 136.1038 = 3.8962 \text{ mm}$

Distance of neutral axis to inner radius $c_i = r_n - r_i = 136.1038\text{-}100 = 36.1038 \text{ mm}$

Distance of neutral axis to outer radius $c_o = r_o - r_n = 180 - 136.1038 = 43.8962 \text{ mm}$

Distance from centroidal axis to force $l = 175 + r_c = 175 + 140 = 315 \text{ mm}$

Applied force $F = 9000 \text{ N}$

Area of cross section $A = 50 \times 80 = 4000 \text{ mm}^2$

Bending moment about centroidal axis $M_b = Fl = 9000 \times 315 = 2835000 \text{ N-mm}$

Direct stress $\sigma_d = \frac{F}{A} = \frac{9000}{4000} = 2.25 \text{ N/mm}^2$ (tensile)

Bending stress at the inner fibre $\sigma_{bi} = \frac{M_b c_i}{Aer_i} = \frac{2835000 \times 36.1038}{4000 \times 3.8962 \times 100}$

$= 65.676 \text{ N/mm}^2$ (tensile)

Bending stress at the outer fibre $\sigma_{bo} = \frac{-M_b c_o}{Aer_o} = \frac{2835000 \times 43.8962}{4000 \times 3.8962 \times 180}$

$= -44.362 \text{ N/mm}^2$ (Compressive)

Combined stress at the inner fibre $\sigma_{ri} = \sigma_d + \sigma_{bi} = 2.25 + 65.676$

$= 67.926 \text{ N/mm}^2$ (tensile)

Combined stress at the outer fibre $\sigma_{ro} = \sigma_d + \sigma_{bo} = 2.25 - 44.362$

$= -42.112 \text{ N/mm}^2$ (Compressive)

Maximum shear stress $\tau_{max} = 0.5\, \sigma_{max} = 0.5 \times 67.926$

$= 33.963 \text{ N/mm}^2$, At the inner fibre

The stress distribution on the critical section is as shown in Fig. 1.6c.

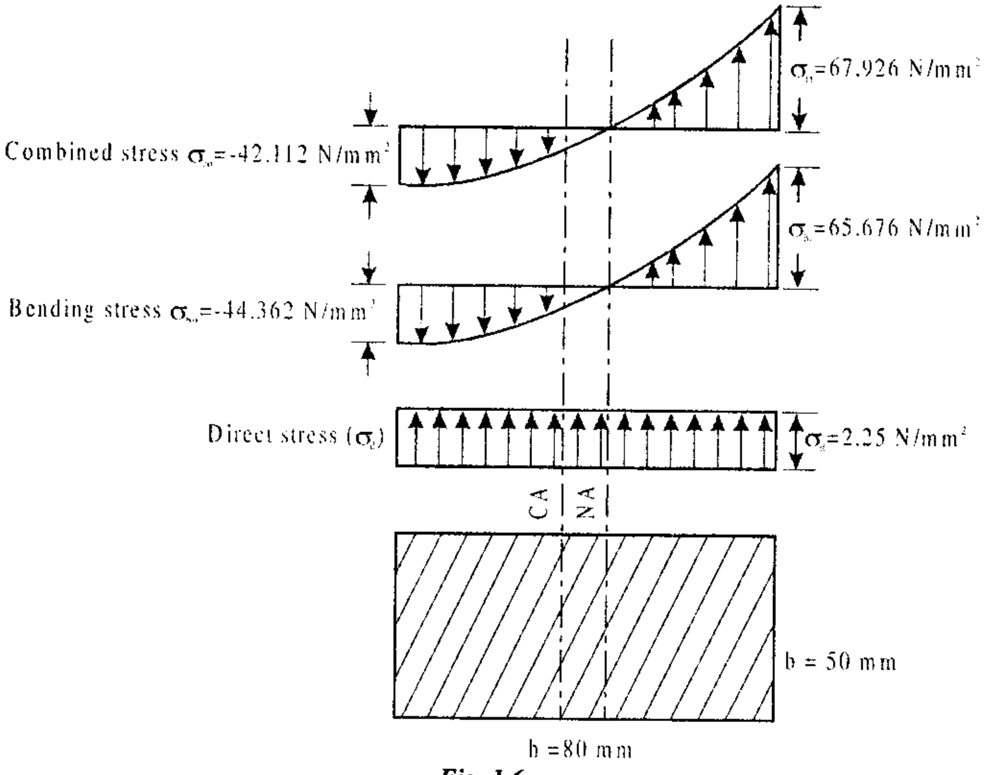


Fig. 1.6c

Exercise : 1.2

The frame of a punch press is shown in Fig. 1.7a. Find the stress in inner and outer surfaces at section A – B of the frame if F = 5000 N

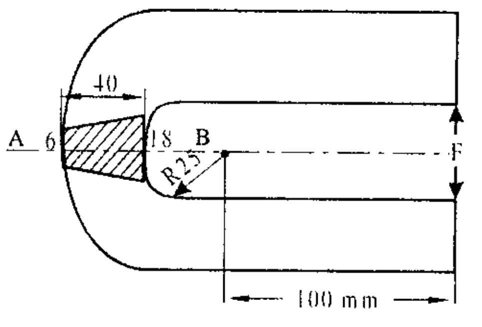


Fig. 1.7a

Solution :

Redraw the critical section as shown in Fig. 1.7b.

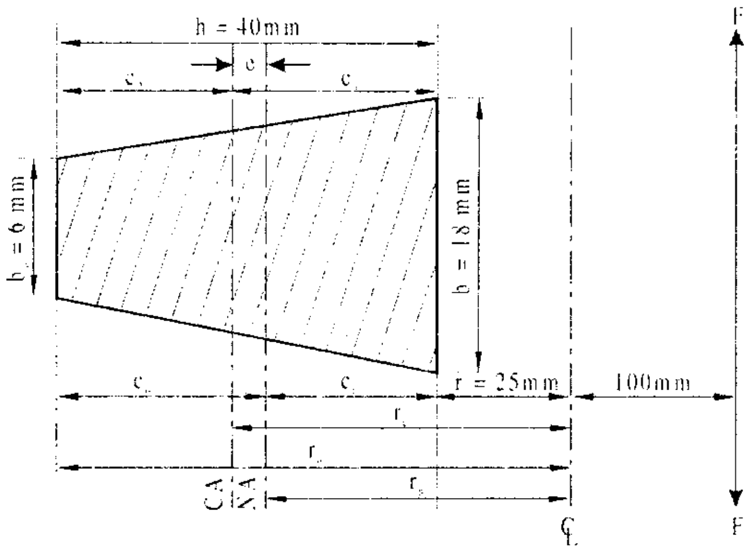


Fig. 1.7b

Inner radius r_i = 25 mm

Outer radius r_o = 25 + 40 = 65 mm

Distance of centroidal axis from inner fibre $c_i = \frac{h}{3}\left(\frac{b_i + 2b_o}{b_i + b_o}\right)$

$$= \frac{40}{3}\left(\frac{18+2\times 6}{18+6}\right) = 16.667 \text{ mm}$$

∴ Radius of centroidal axis $r_c = r_i + c_i = 25 + 16.667 = 41.667$ mm

Radius of neutral axis

$$r_n = \frac{\frac{1}{2}h(b_i + b_o)}{\frac{(b_i r_o - b_o r_i)}{h} ln\frac{(r_o)}{r_i} - (b_i - b_o)} \qquad .....25.62$$

$$= \frac{\frac{1}{2}\times 40(18+6)}{\frac{(18\times 65 - 6\times 25)}{40} ln\frac{(65)}{25} - (18-6)} = 38.8175 \text{ mm}$$

Distance of neutral axis to centroidal axis $e = r_c - r_n$

$= 41.667 - 38.8175 = 2.8495$ mm

Distance of neutral axis to inner radius $c_i = r_n - r_i$

$= 38.8175 - 25 = 13.8175$ mm

Distance of neutral axis to outer radius $c_o = r_o - r_n = 65 - 38.8175 = 26.1825$ mm

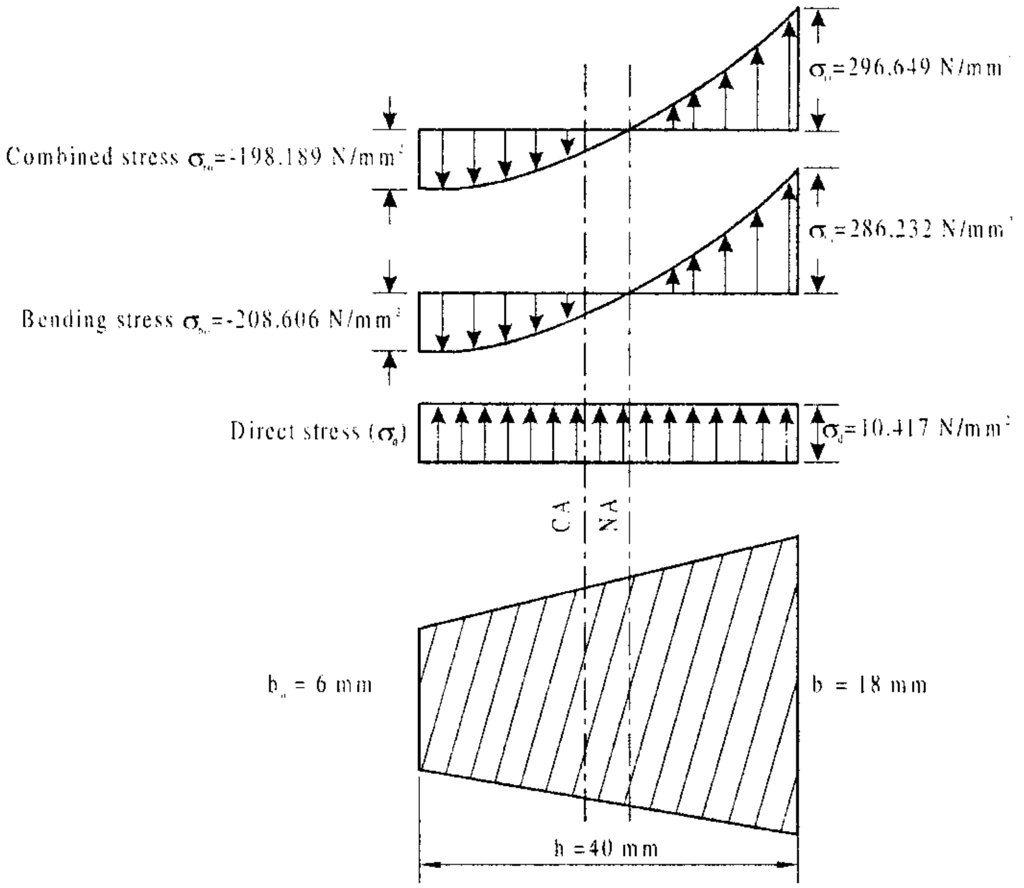


Fig. 1.7c

Area of cross section $A = \frac{1}{2}(b_i + b_o)\,h = \frac{1}{2}(18 + 6)\,40 = 480\ \text{mm}^2$

Applied force $F = 5000\ \text{N}$

Distance from centroidal axis to force $l = 100 + r_c = 100 + 41.667 = 141.667\ \text{mm}$

Bending moment about centroidal axis $M_b = Fl = 5000 \times 141.667 = 708335\ \text{N.mm}$

Direct stress $\sigma_d = \frac{F}{A} = \frac{5000}{480} = 10.417\ \text{N/mm}^2$ (tensile)

Bending stress at the inner fibre $\sigma_{bi} = \frac{M_b c_i}{Aer_i}$

$$= \frac{708335 \times 13.8175}{480 \times 2.8495 \times 25} = 286.232\ \text{N/mm}^2 \text{ (tensile)}$$

Bending stress at the outer fibre $\sigma_{bo} = -\frac{M_b c_o}{Aer_o} = \frac{708335 \times 26.1825}{480 \times 2.8495 \times 65} = -208.606\ \text{N/mm}^2$

(Compressive)

Combined stress at the inner fibre $\sigma_{ri} = \sigma_d + \sigma_{bi} = 10.417 + 286.232$

$= 296.649$ N/mm^2 (tensile)

Combined stress at the outer fibre $\sigma_{ro} = \sigma_d + \sigma_{bo} = 10.417 - 208.606$

$= -198.189$ N/mm^2 (Compressive)

Maximum shear stress $\tau_{max} = 0.5\,\sigma_{max} = 0.5 \times 296.649$

$= 148.3245$ N/mm^2, At the inner fibre.

The stress distribution on the critical section is as shown in Fig. 1.7c

Example : 1.3

Figure 1.8a shows a frame of a punching machine and its various dimensions. Determine the maximum stress in the frame, if it has to resist a force of 85 kN.

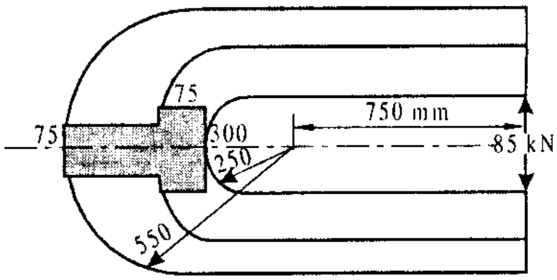


Fig. 1.8a

Solution :

Redraw the critical section as shown in Fig. 1.8b

Inner radius of curved beam $r_i = 250$ mm

Outer radius of curved beam $r_o = 550$ mm

Radius of neutral axis $$r_n = \frac{A}{b_i \ln\left(\frac{r_i + a_i}{r_i}\right) + b_2 \ln\left(\frac{r_o - a_o}{r_i + a_i}\right) + b_o \ln\left(\frac{r_o}{r_o - a_o}\right)} \quad \text{..... 25.63}$$

$a_i = 75$ mm; $b_i = 300$ mm; $b_2 = 75$ mm

$a_o = 0$; $b_o = 0$; $A = a_1 + a_2 = 75 \times 300 + 75 \times 225 = 39375$ mm^2

$$\therefore\ r_n = \frac{39375}{300 \ln\left(\frac{250+75}{250}\right) + 75 \ln\left(\frac{550-0}{250+75}\right) + 0} = 333.217 \text{ mm}$$

Let AB be the ref. line

$$\therefore\ \bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(75 \times 300)\frac{(75)}{2} + (75 \times 225)\left(75 + \frac{225}{2}\right)}{39375}$$

$= 101.785$ mm

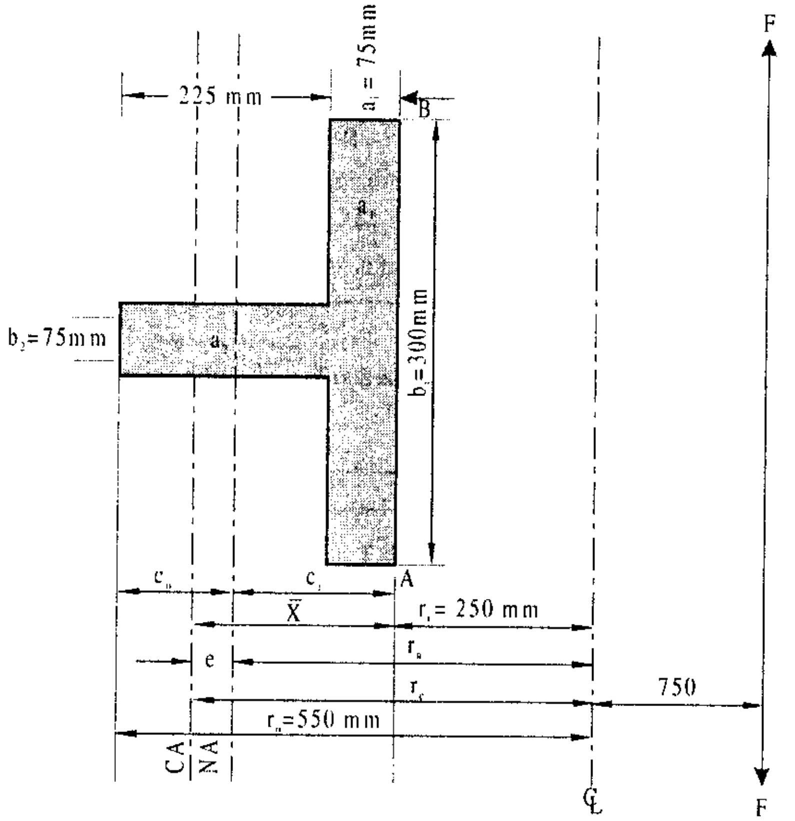


Fig. 1.8b

$\therefore$ Radius of centroidal axis $r_c = r_i + \bar{x} = 250 + 101.785 = 351.785$ mm

Distance of neutral axis to centroidal axis, $e = r_c - r_n = 351.785 - 333.217 = 18.568$ mm

Distance of neutral axis to inner radius $c_i = r_n - r_i = 333.217 - 250 = 83.217$ mm

Distance of neutral axis to outer radius $c_o = r_o - r_n = 550 - 333.217 = 216.783$ mm

Distance from centroidal axis to force $l = 750 + r_c = 750 + 351.785 = 1101.785$ mm

Applied force $F = 85\text{ kN} = 85{,}000\text{ N}$

Bending moment about centroidal axis $M_b = Fl = 85{,}000 \times 1101.785 = 93651725$ Nmm

Direct stress $\sigma_d = \dfrac{F}{A} = \dfrac{85000}{39375} = 2.16\text{ N/mm}^2$ (tensile)

Bending stress at the inner fibre $\sigma_{bi} = \dfrac{M_b c_i}{A e r_i} = \dfrac{93651725 \times 83.217}{39375 \times 18.568 \times 250}$

$= 42.64\text{ N/mm}^2$ (tensile)

Bending stress at the outer fibre $\sigma_{bo} = -\dfrac{M_b C_o}{A e r_o} = \dfrac{93651725 \times 216.783}{39375 \times 18.568 \times 550}$

$= -50.49\text{ N/mm}^2$ (Compressive)

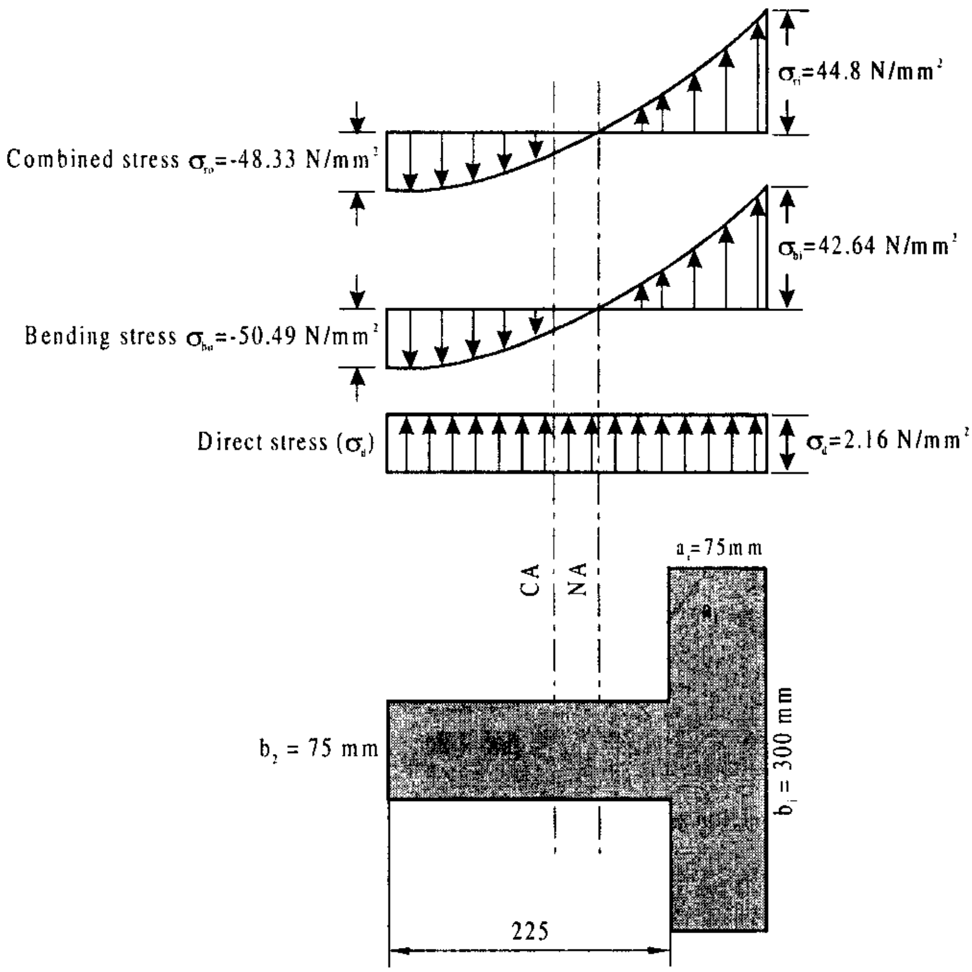


Fig. 1.18c

Combined stress at the inner fibre $\sigma_{ri} = \sigma_d + \sigma_{bi} = 2.16 + 42.64$

$= 44.8\ \text{N/mm}^2$ (tensile)

Combined stress at the outer fibre $\sigma_{ro} = \sigma_d + \sigma_{bo} = 2.16 - 50.49$

$= -48.33\ \text{N/mm}^2$ (compressive)

Maximum shear stress $\tau_{max} = 0.5\,\sigma_{max} = 0.5 \times 48.33 = 24.165\ \text{N/mm}^2$

At the outer fibre

The stress distribution on the critical section is as shown in Fig. 1.18c

Example : 1.4

Determine the stresses at point A and B of the split ring shown in Fig. 1.9a

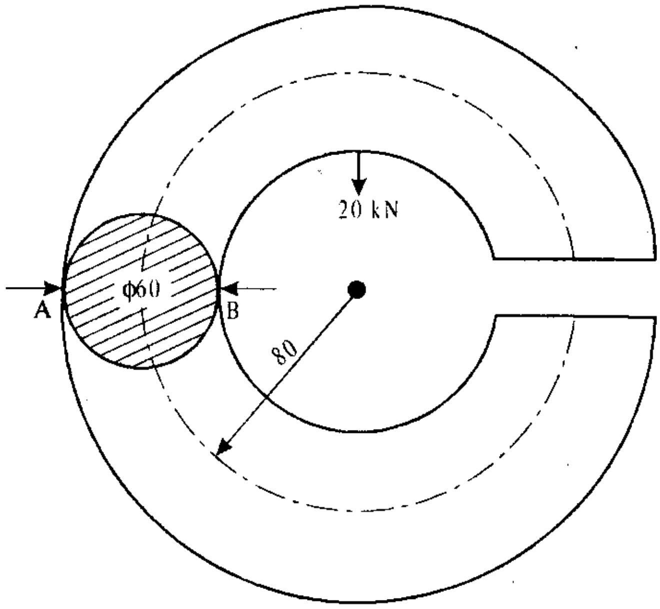


Fig. 1.9a

Solution :

Redraw the critical section as shown in Fig. 1.9b

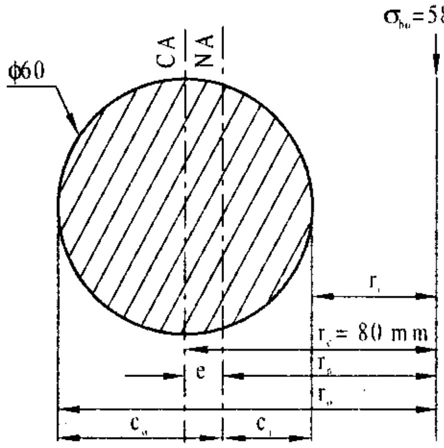


Fig. 1.9b

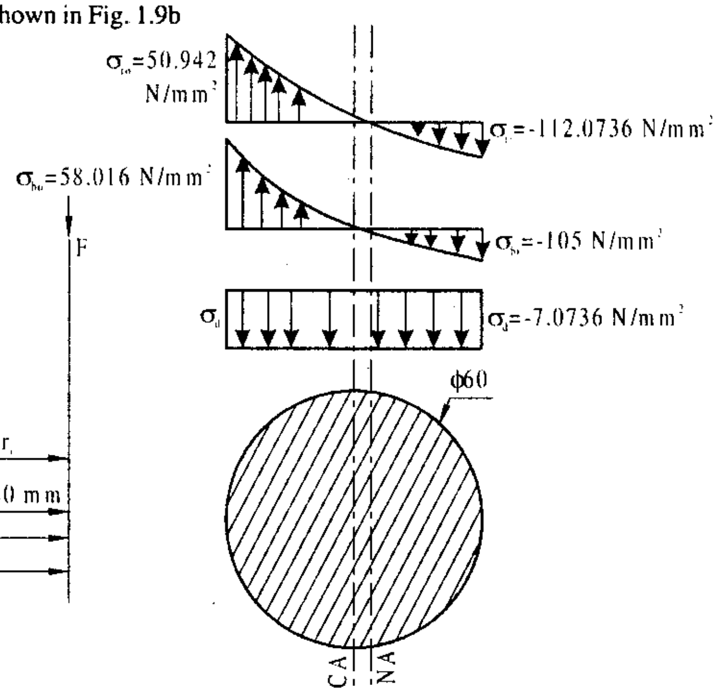


Fig. 1.9c

Radius of centroidal axis $r_c = 80\text{mm}$

Inner radius $r_i = 80 - \frac{60}{2} = 50 \text{ mm}$

Outer radius $r_o = 80 + \frac{60}{2} = 110 \text{ mm}$

Radius of neutral axis $r_n = \frac{\left(\sqrt{r_o} + \sqrt{r_i}\right)^2}{4}$ 25.60

$= \frac{\left(\sqrt{110} + \sqrt{50}\right)^2}{4} = 77.081 \text{ mm}$

Applied force $F = 20 \text{ kN} = 20{,}000 \text{ N}$ (Compressive)

Area of cross - section $A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 60^2 = 2827.433 \text{ mm}^2$

Distance from centroidal axis to force $l = r_c = 80 \text{ mm}$

Bending moment about centroidal axis $M_b = Fl = 20{,}000 \times 80 = 16 \times 10^5 \text{ Nmm}$

Distance of neutral axis to centroidal axis $e = r_c - r_n = 80 - 77.081 = 2.919 \text{ mm}$

Distance of neutral axis to inner radius $c_i = r_n - r_i = 77.081 - 50 = 27.081 \text{ mm}$

Distance of neutral axis to outer radius $c_o = r_o - r_n = 110 - 77.081 = 32.919 \text{ mm}$

Direct Stress $\sigma_d = -\frac{F}{A} = -\frac{20000}{2827.433} = -7.0736 \text{ N/mm}^2$ (Comp.)

Bending stress at the inner fibre $\sigma_{bi} = -\frac{M_b c_i}{A e r_i} = \frac{-16 \times 10^5 \times 27.081}{2827.433 \times 2.919 \times 50}$

$= -105 \text{ N/mm}^2$ (compressive)

Bending stress at the outer fibre $\sigma_{bo} = \frac{M_b c_o}{A e r_o} = \frac{16 \times 10^5 \times 32.919}{2827.433 \times 2.919 \times 110}$

$= 58.016 \text{ N/mm}^2$ (tensile)

Combined stress at the inner fibre (at B) $\sigma_{ri} = \sigma_d + \sigma_{bi} = -7.0736 - 105.00$

$= -112.0736 \text{ N/mm}^2$ (Compressive)

Combined stress at the outer fibre (at A) $\sigma_{ro} = \sigma_d + \sigma_{bo} = -7.0736 + 58.016$

$= 50.9424 \text{ N/mm}^2$ (tensile)

Maximum shear stress $\tau_{max} = 0.5\,\sigma_{max} = 0.5 \times 112.0736 = 56.0368 \text{ N/mm}^2$ at B

The stress distribution on the critical section is as shown in Fig. 1.9c.

Example : 1.5

Compute the combined stresses at the inner and outer fibres in the critical cross section of a crane hook which is required to lift loads up to 25 kN. The hook has trapezoidal cross-section with parallel sides 60 mm and 30 mm, the distance between them being 90 mm The inner radius of the hook is 100 mm. The load line is nearer to the inner surface of the hook by 25 mm than the centre of curvature at the critical section. What will be the stresses at the inner and outer fibre, if the beam is treated as straight beam for the given load. (VTU, Dec'06/Jan'07)

Solution :

The given crane hook is as shown in Fig. 1.10a

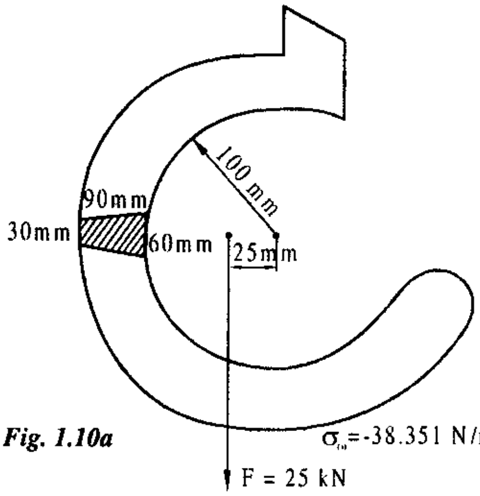


Fig. 1.10a

(a) Beam is treated as curved beam

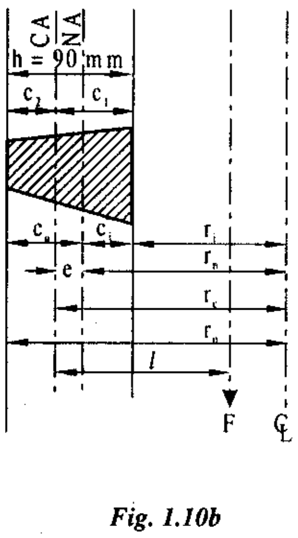


Fig. 1.10b

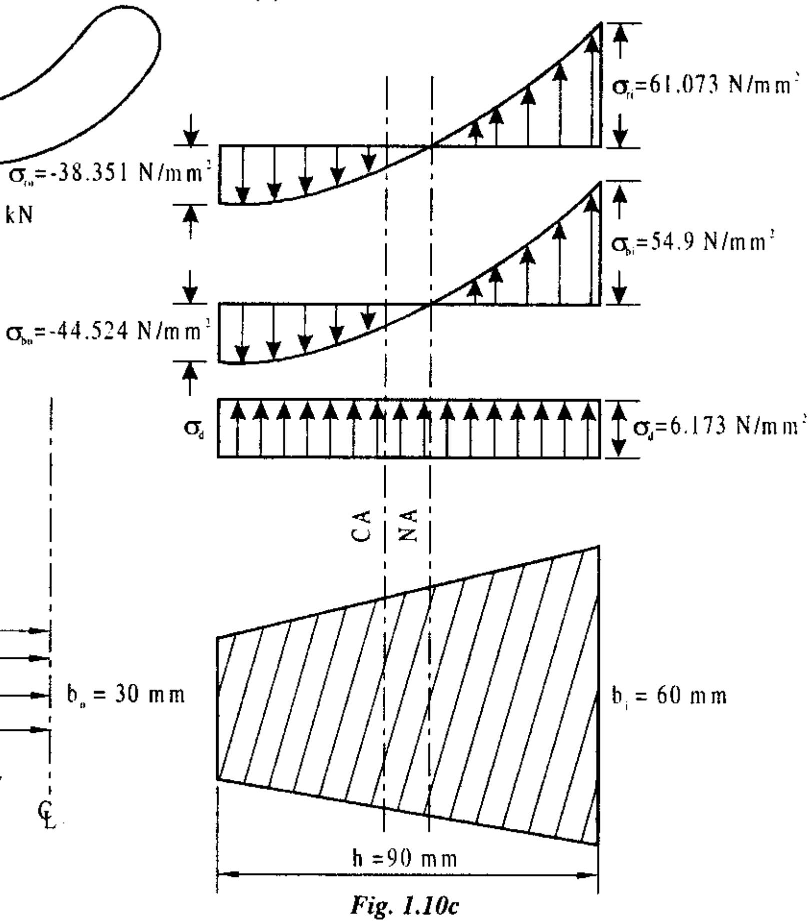


Fig. 1.10c

Inner radius $r_i = 100$ mm

Outer radius $r_o = 100 + 90 = 190$ mm

Distance of centroidal axis from the inner fibre

$$c_i = \frac{h}{3}\left(\frac{b_i + 2b_o}{b_i + b_o}\right)$$

$$= \frac{90}{3}\left(\frac{60 + 2\times 30}{60 + 30}\right) = 40 \text{ mm}$$

∴ Radius of centroidal axis $r_c = r_i + c_i = 100 + 40 = 140$ mm

Radius of neutral axis

$$r_n = \frac{\frac{1}{2}h(b_i + b_o)}{\left(\frac{b_i r_o - b_o r_i}{h}\right) ln\left(\frac{r_o}{r_i}\right) - (b_i - b_o)} \quad \text{---- 25.62}$$

$$= \frac{\frac{1}{2}\times 90\times(60+30)}{\left(\frac{60\times 190 - 30\times 100}{90}\right) ln\left(\frac{190}{100}\right) - (60-30)}$$

$$= 135.42 \text{ mm}$$

Distance of neutral axis from centroidal axis $e = r_c - r_n = 140 - 135.42 = 4.58$ mm

Area of cross section $A = \frac{1}{2}(b_i + b_o)\,h = \frac{1}{2}(60 + 30)\,90 = 4050 \text{ mm}^2$

Distance of neutral axis to inner radius $c_i = r_n - r_i = 135.42 - 100 = 35.42$ mm

Distance of neutral axis to outer radius $c_o = r_o - r_n = 190 - 135.42 = 54.58$ mm

Applied force $F = 25$ kN $= 25{,}000$N

Distance from centroidal axis to force $l = r_c - 25 = 140 - 25 = 115$ mm

Bending moment about centroidal axis $M_b = Fl = 25{,}000 \times 115 = 2875000$ Nmm

Direct stress $\sigma_d = \frac{F}{A} = \frac{25{,}000}{4050} = 6.173 \text{ N/mm}^2$ (tensile)

Bending stress at the inner fibre $\sigma_{bi} = \frac{M_b c_i}{A e r_i} = \frac{2875000\times 35.42}{4050\times 4.58\times 100}$

$= 54.9 \text{ N/mm}^2$ (tensile)

Bending stress at the outer fibre $\sigma_{bo} = -\frac{M_b c_o}{A e r_o} = \frac{2875000\times 54.58}{4050\times 4.58\times 190}$

$= -44.524 \text{ N/mm}^2$ (compressive)

Combined stress at the inner fibre $\sigma_{ri} = \sigma_d + \sigma_{bi} = 6.173 + 54.9 = 61.073$ N/mm² (tensile)

Combined stress at the outer fibre $\sigma_{ro} = \sigma_d + \sigma_{bo} = 6.173 - 44.524$

$= -38.351$ N/mm² (compressive)

Maximum shear stress $\tau_{max} = 0.5\,\sigma_{max} = 0.5 \times 61.072$

$= 30.5365$ N/mm² at the inner fibre

The stress distribution on the curved beam is as shown in Fig. 1.10c.

(b) Beam is treated as straight beam

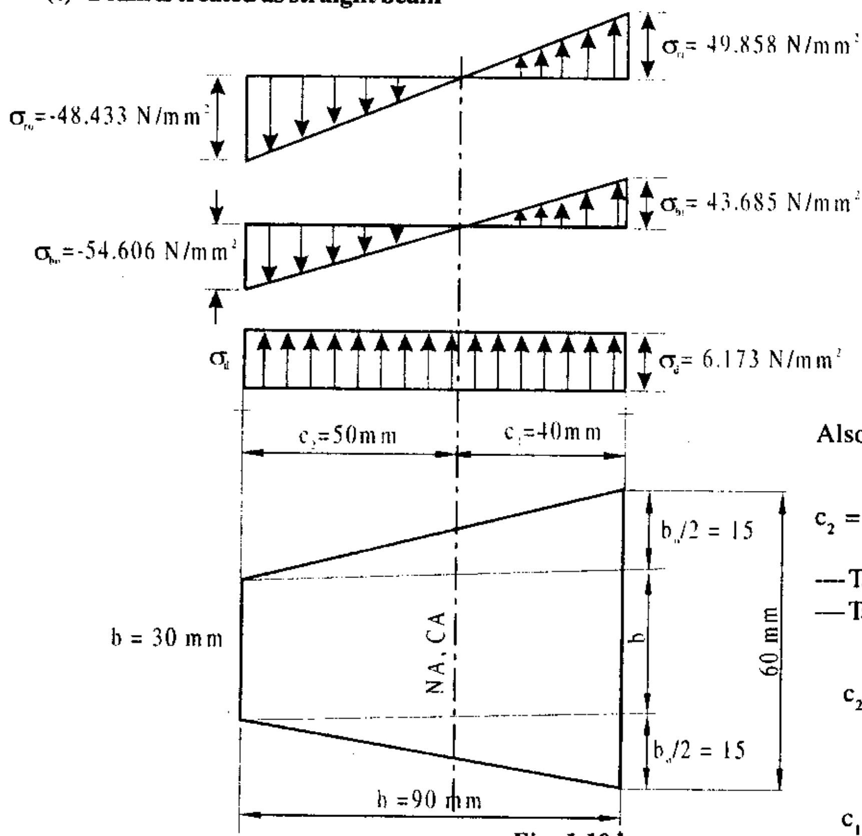


Fig. 1.10d

$b = 30$ mm;

$b_o = 60 - 30 = 30$ mm

$h = 90$ mm

$c_1 = 40$ mm

$c_2 = 90 - 40 = 50$ mm

$A = 4050$ mm²

$M_b = 28750000$ Nmm

Also

$$c_2 = \frac{(3b + 2b_0)h}{3(2b + b_0)}$$

— Table 2.6 (Old DDHB Vol. I)
— Table 2.7 (New DDHB Vol. I)

$$c_2 = \frac{(3 \times 30 + 2 \times 30)90}{3(2 \times 30 + 30)}$$

$= 50$ mm

$c_1 = 90 - 50 = 40$ mm

From Table 2.6 (vol.I Old DDHB) or Table 2.7 (Vol.I New DDHB)

Moment of Inertia
$$I = \frac{(6b^2 + 6bb_o + b_0^2)h^3}{36[2b + b_o]} = 2632500 \text{ mm4}$$

$$= \frac{(6 \times 30^2 + 6 \times 30 \times 30 + 30^2)90^3}{36[2 \times 30 + 30]} = 2632500 \text{ mm}^4$$

Direct stress
$$\sigma_d = \frac{F}{A} = \frac{25{,}000}{4050} = 6.173 \text{ N/mm}^2 \text{ (tensile)}$$

Bending stress at the inner fibre $\sigma_{bi} = \frac{M_b c_1}{I} = \frac{2875000 \times 40}{2632500} = 43.685$ N/mm^2 (tensile)

Bending stress at the outer fibre $\sigma_{bo} = -\frac{M_b c_2}{I} = \frac{2875000 \times 50}{2632500}$

$= -54.606$ N/mm^2 (Compressive)

$\therefore$ Combined stress at the inner fibre $\sigma_{ri} = \sigma_d + \sigma_{bi} = 6.173 + 43.685 = 49.858$ N/mm^2

Combined stress at the outer fibre $\sigma_{ro} = \sigma_d + \sigma_{bo} = 6.173 - 54.606$

$= -48.433$ N/mm^2 (compressive)

The stress distribution on the straight beam is as shown in Fig. 1.10d.

Example : 1.6

The section of a crane hook is rectangular in shape whose width is 30 mm and depth is 60 mm. The centre of curvature of the section is at a distance of 125 mm from the inside section and the load line is 100 mm from the same point. Find the capacity of the hook if the allowable stress in tension is 75 N/mm².

Solution :

The given crane hook is as shown in Figure 1.11a

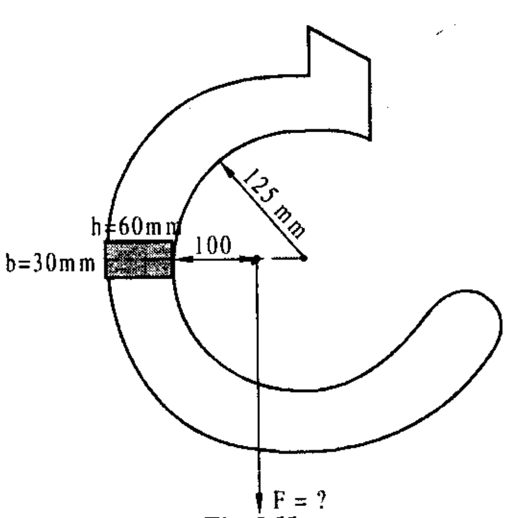


Fig. 1.11a

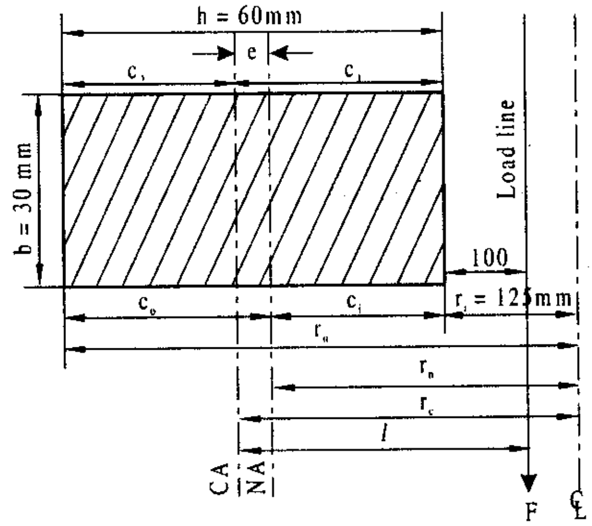


Fig. 1.11b

Redraw the critical section as shown in Fig. 1.11b

Inner radius $r_i = 125$ mm

Outer radius $r_o = 125 + 60 = 185$ mm.

Radius of centroidal axis. $r_c = 125 + \frac{60}{2} = 155$ mm.

Radius of neutral axis $r_n = \dfrac{h}{ln\left(\dfrac{r_o}{r_i}\right)}$ ---25.61

$$= \frac{60}{ln\left(\frac{185}{125}\right)} = 153.045 \text{ mm}$$

Distance of centroidal axis from neutral axis $e = r_c - r_n = 155 - 153.045 = 1.955$ mm

Distance of neutral axis to inner radius $c_i = r_n - r_i = 153.045 - 125 = 28.045$ mm

Distance of neutral axis to outer radius $c_o = r_o - r_n = 185 - 153.045 = 31.955$ mm

Area of cross section $A = bh = 30 \times 60 = 1800 \text{ mm}^2$

Distance from centroidal axis to force $l = r_c - 25 = 155 - 25 = 130$ mm

Bending moment about centroidal axis $M_b = Fl = 130F$

Maximum tensile stress is at the inner fibre.

∴ Combined stress at the inner fibre $\sigma_{ri} = \dfrac{F}{A} + \dfrac{M_b c_i}{Aer_i}$

$$\text{i.e., } 75 = \frac{F}{1800} + \frac{130F \times 28.045}{1800 \times 1.955 \times 125}$$

$$\therefore F = 8480.4\text{N} = \text{Capacity of the hook.}$$

Example : 1.7

Design a steel crane hook to have a capacity of 100 kN. Assume factor of safety (FS) = 2 and trapezoidal section.

Data :

Load capacity F = 100 kN = 10^5 N; Trapezoidal section; FS = 2

Solution :

Approximately 1kgf = 10N

$\therefore 10^5\text{N} \approx 10{,}000 \text{ kgf} \approx 10\text{t}$

Select the standard crane hook dimensions from table 25.3 when safe load = 10t and steel (MS)

$\therefore c = 119$ mm; $Z = 14$ mm; $M = 71$ mm and $H = 111$ mm

$\therefore b_i = M = 71$ mm

$b_o = 2Z = 2 \times 14 = 28$ mm

$r_i = \dfrac{c}{2} = \dfrac{119}{2} = 59.5$ mm

$H = 111 \text{ mm} = h$

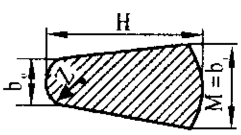


Fig. 1.12a

Assume the load line passes through the centre of hook. Draw the critical section as shown in Fig. 1.12b.

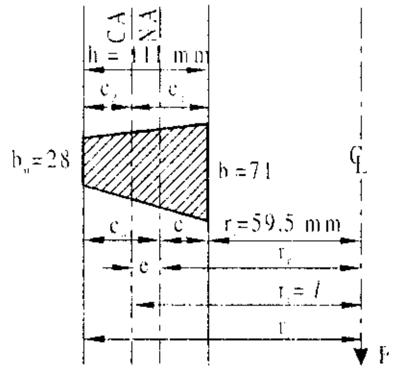


Fig. 1.12b

Inner radius $\quad r_i = 59.5\,\text{mm}$

Outer radius $\quad r_o = 59.5 + 111 = 170.5\,\text{mm}.$

$$\text{Radius of neutral axis } r_n = \frac{\frac{1}{2}h(b_i + b_o)}{\left(\frac{b_i r_o - b_o r_i}{h}\right) ln\left(\frac{r_o}{r_i}\right) - (b_i - b_o)} \qquad \text{----25.62}$$

$$= \frac{\frac{1}{2} \times 111 \times (71 + 28)}{\left(\frac{71 \times 170.5 - 28 \times 59.5}{111}\right) ln\left(\frac{170.5}{59.5}\right) - (71 - 28)}$$

$$= 98.095\,\text{mm}$$

$$\text{Distance of centroidal axis from inner fibre } c_1 = \frac{h}{3}\left(\frac{b_i + 2b_o}{b_i + b_o}\right)$$

$$= \frac{111}{3}\left(\frac{71 + 2 \times 28}{71 + 28}\right) = 47.465\,\text{mm}$$

$$\therefore \text{ Radius of centroidal axis } r_c = r_i + c_1 = 47.465 + 59.5 = 106.965\,\text{mm}$$

$$e = r_c - r_n = 106.965 - 98.095 = 8.87\,\text{mm}$$

$$c_i = r_n - r_i = 98.095 - 59.5 = 38.595\,\text{mm}$$

$$c_o = r_o - r_n = 170.5 - 98.095 = 72.405\,\text{mm}$$

$$l = r_c - 106.965\,\text{mm}$$

$$M_b = Fl = 10^5 \times 106.965 = 106.965 \times 10^5\,\text{Nmm}$$

$$A = \frac{1}{2}h(b_i + b_o) = \frac{1}{2} \times 111\,(71 + 28)$$

$$= 5494.5\ \text{mm}^2$$

Direct stress $\sigma_d = \frac{F}{A} = \frac{10^5}{5494.5} = 18.2\ \text{N/mm}^2$ (tensile)

Bending stress at the inner fibre $\sigma_{bi} = \frac{M_b c_i}{Aer_i}$

$$= \frac{106.965 \times 10^5 \times 38.595}{5494.5 \times 8.87 \times 59.5} = 142.365\ \text{N/mm}^2\ \text{(tensile)}$$

Bending stress at the outer fibre $\sigma_{bo} = -\frac{M_b c_o}{Aer_o} - \frac{106.965 \times 10^5 \times 72.405}{5494.5 \times 8.87 \times 170.5}$

$$= -93.2\ \text{N/mm}^2\ \text{(Compressive)}$$

$\therefore$ Combined stress at the inner fibre $\sigma_{ri} = \sigma_d + \sigma_{bi} = 18.2 + 142.365 = 160.565\ \text{N/mm}^2$ (tensile)

Combined stress at the outer fibre $\sigma_{ro} = \sigma_d + \sigma_{bo}$

$$= 18.2 - 93.2 = -75\ \text{N/mm}^2\ \text{(Compressive)}$$

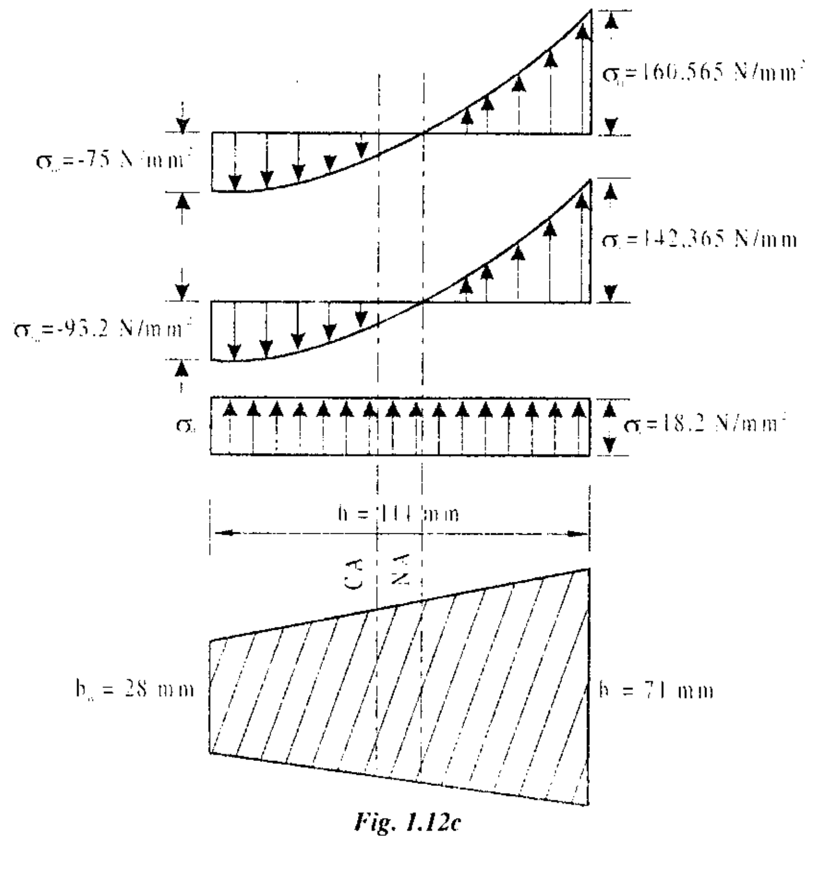


Fig. 1.12c

Maximum shear stress $\tau_{max} = 0.5\sigma_{max} = 0.5 \times 160.565$

$= 80.2825\ N/mm^2$. At the inner fibre

The stress distribution at the critical section of beam is as shown in Fig. 1.12.c.

Example : 1.8

Determine the force F such that the maximum tensile stress in the clamp is not to exceed 140 N/mm²

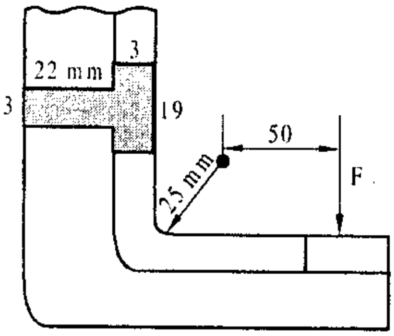


Fig. 1.13a

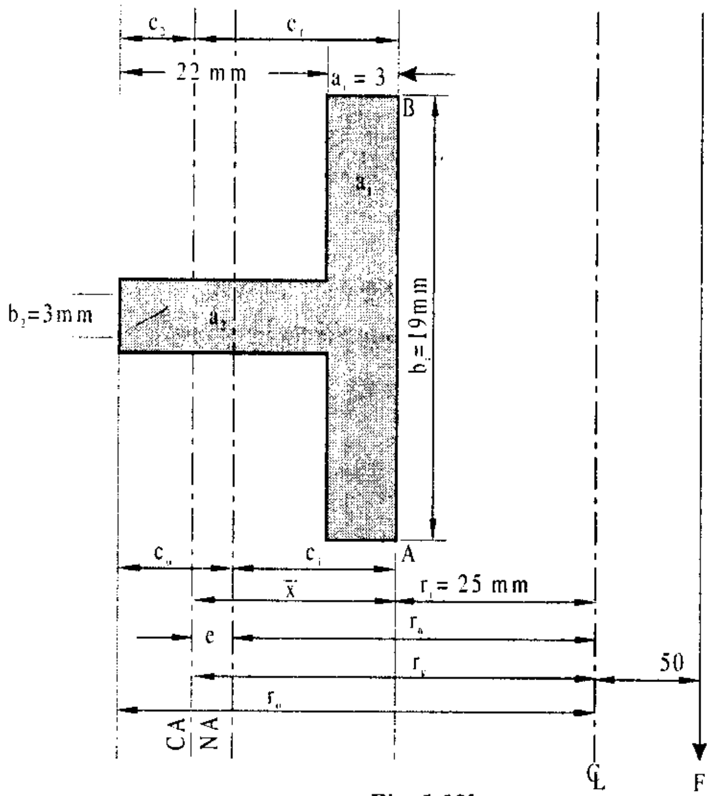


Fig. 1.13b

Solution :

Redraw the critical section as shown in Fig. 1.13b

Inner radius $r_i = 25mm$

Outer radius $r_o = 25 + 3 + 22 = 50\ mm.$

$$\text{Radius of neutral axis } r_n = \frac{A}{b_i ln\left(\frac{r_i + a_i}{r_i}\right) + b_2 ln\left(\frac{r_o - a_o}{r_i + a_i}\right) + b_o ln\left(\frac{r_o}{r_o - a_o}\right)} \qquad \text{---- 25.63}$$

$a_i = 3\ mm$; $b_i = 19\ mm$; $b_2 = 3\ mm$; $a_o = 0\ mm$; $b_o = 0$

$A = a_1 + a_2 = 19 \times 3 + 22 \times 3 = 123\ mm^2$

$$\therefore\ r_n = \frac{123}{19 ln\left(\frac{25+3}{25}\right) + 3 ln\left(\frac{50-0}{25+3}\right) + 0} = 31.5976 \text{mm}.$$

Let AB be the reference line

$$\therefore\ \bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(3\times19)\left(\frac{3}{2}\right) + (22\times3)\left(3+\frac{22}{2}\right)}{123} = 8.2 \text{ mm}$$

$$\therefore\ \text{Radius of centroidal axis } r_c = r_i + \bar{x} = 25 + 8.2 = 33.2 \text{ mm}$$

$$e = r_c - r_n = 33.2 - 31.5976 = 1.6024 \text{ mm}$$

$$c_i = r_n - r_i = 31.5976 - 25 = 6.5976 \text{ mm}$$

$$c_o = r_o - r_n = 50 - 31.5976 = 18.4024 \text{ mm}$$

$$l = r_c + 50 = 33.2 + 50 = 83.2 \text{ mm}$$

$$M_b = Fl = 83.2F$$

Maximum tensile stress is at the inner fibre

$\therefore$ Combined maximum stress at the inner fibre

$$\sigma_{r_i} = \frac{F}{A} + \frac{M_b c_i}{A e r_i}$$

$$\text{i.e., } 140 = \frac{F}{123} + \frac{83.2F \times 6.5976}{123 \times 1.6024 \times 25}$$

$$\therefore\ F = 1171.2\text{N}$$

Example : 1.9

Determine the maximum tensile stress and maximum shear stress of the component shown in Fig. 1.14a and indicate the location.

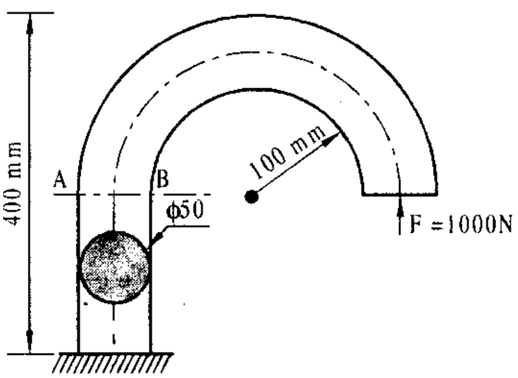


Fig. 1.14a

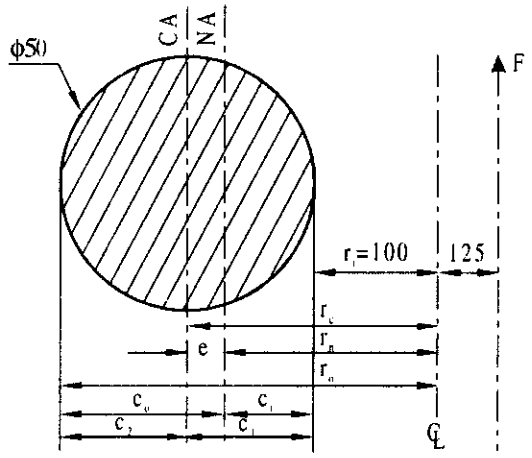


Fig. 1.14b

Solution :

Re draw the critical section as shown in Fig. 1.14b

Inner radius $r_i = 100\text{mm}$

Outer radius $r_o = 100+50 = 150\text{mm}.$

Radius of neutral axis $r_n = \dfrac{\left(\sqrt{r_o}+\sqrt{r_i}\right)^2}{4} = \dfrac{\left(\sqrt{150}+\sqrt{100}\right)^2}{4}$ ---- 25.62

$= 123.737 \text{ mm}$

$e = r_c - r_n = 125 - 123.737 = 1.263 \text{ mm}$

$c_i = r_n - r_i = 123.737 - 100 = 23.737 \text{ mm}$

$c_o = r_o - r_n = 150 - 123.737 = 26.263 \text{ mm}$

$A = \dfrac{\pi}{4}d^2 = \dfrac{\pi}{4} \times 50^2 = 1963.495 \text{ mm}^2$

$l = r_c + 125 = 125 + 125 = 250 \text{ mm}$

$M_b = Fl = 1000 \times 250 = 25 \times 10^4 \text{ Nmm}$

Direct stress $\sigma_d = \dfrac{F}{A} = \dfrac{1000}{1963.495} = 0.5093 \text{ N/mm}^2$ (tensile)

Bending stress at the inner fibre $\sigma_{bi} = \dfrac{M_b c_i}{A e r_i} = \dfrac{25\times10^4\times23.737}{1963.495\times1.263\times100}$

$= 23.9294 \text{ N/mm}^2$ (tensile)

Bending stress at the outer fibre $\sigma_{bo} = -\dfrac{M_b c_o}{A e r_o} = -\dfrac{25\times10^4\times26.263}{1963.495\times1.263\times150}$

$= 17.6506 \text{ N/mm}^2$ (Compressive)

∴ Combined stress at the inner fibre $\sigma_{ri} = \sigma_d + \sigma_{bi} = 0.5093 + 23.9294$

$= 24.4387 \text{ N/mm}^2$ (tensile)

Combined stress at the outer fibre $\sigma_{ro} = \sigma_d + \sigma_{bo} = 0.5093 - 17.6506$

$= -17.1413 \text{ N/mm}^2$ (Compressive)

∴ Maximum tensile stress $= 24.4387 \text{ N/mm}^2$ at B

Maximum shear stress $\tau_{max} = 0.5\,\tau_{max} = 0.5 \times 24.4387 = 12.2193 \text{ N/mm}^2$ at B

The stress distribution at the critical section of the machine member is as shown in Fig. 1.14c.

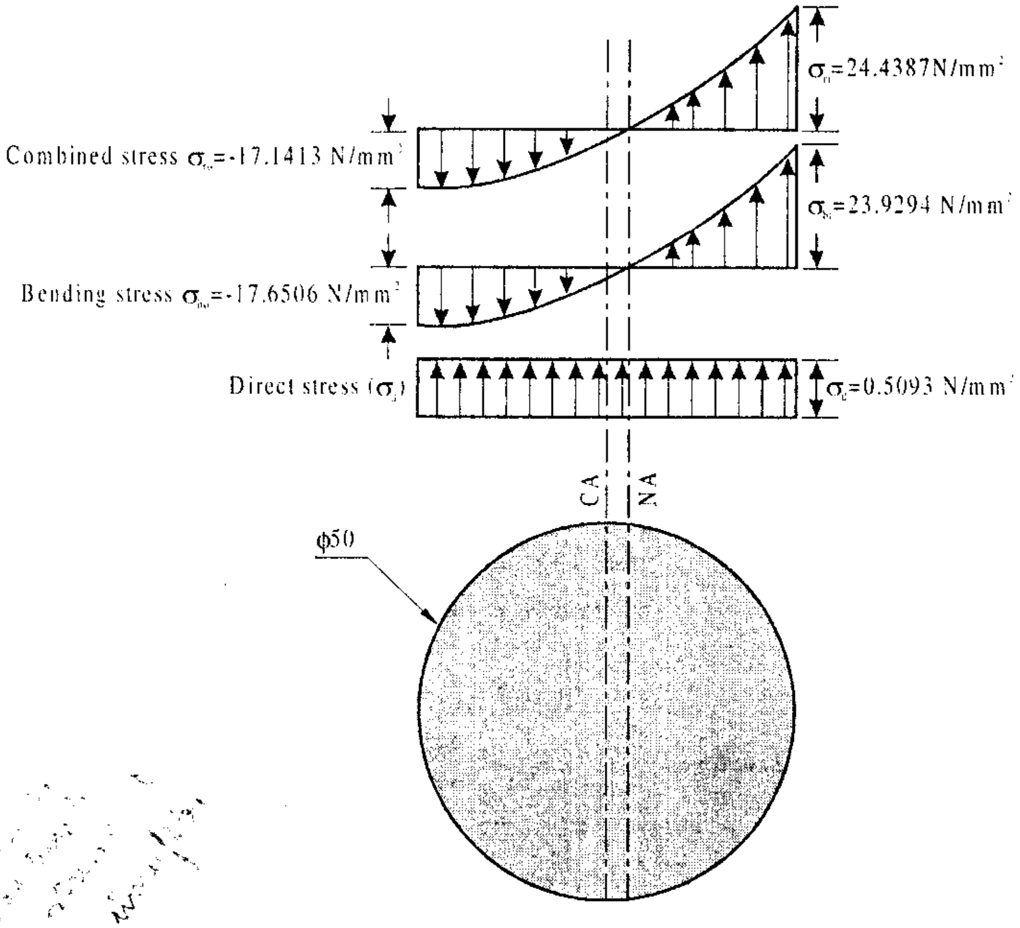


Fig. 1.14c

Example : 1.10

Determine the maximum tensile stress of the machine component shown in Fig. 1.15a and indicate the location.

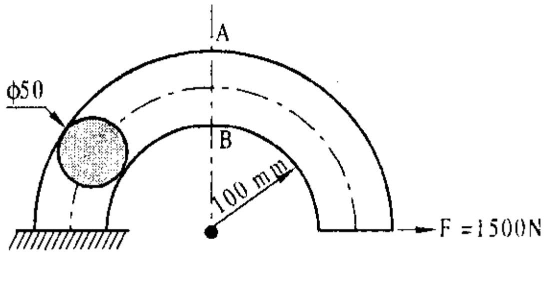


Fig. 1.15a

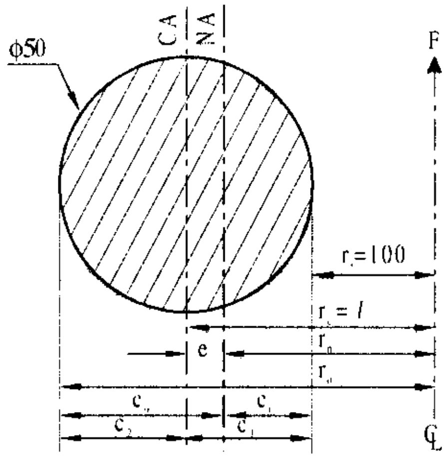


Fig. 1.15b

Solution :

Re draw the critical section as shown in Fig. 1.15b

Inner radius $r_i = 100 \text{mm}$

Outer radius $r_o = 100+50 = 150 \text{ mm}.$

Radius of centroidal axis $r_c = 100 + \frac{50}{2} = 125 \text{ mm}$

Radius of neutral axis $r_n = \frac{\left(\sqrt{r_o} + \sqrt{r_i}\right)^2}{4}$ ---- 25.60

$$= \frac{\left(\sqrt{150} + \sqrt{100}\right)^2}{4} = 123.737 \text{ mm}$$

$$e = r_c - r_n = 125 - 123.737 = 1.263 \text{ mm}$$

$$c_i = r_n - r_i = 123.737 - 100 = 23.737 \text{ mm}$$

$$c_o = r_o - r_n = 150 - 123.737 = 26.263 \text{ mm}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 50^2 = 1963.495 \text{ mm}^2$$

$$l = r_c = 125 \text{mm}$$

$$M_b = Fl = 1500 \times 187500 \text{Nmm}$$

Maximum tensile stress is at the inner fibre.

∴ Combined maximum stress at the inner fibre

$$\sigma_n = \text{Direct stress + Bending stress} = \sigma_d + \sigma_{bi}$$

$$= \frac{F}{A} + \frac{M_b c_i}{A e r_i} = \frac{1500}{1963.495} + \frac{187500 \times 23.737}{1963.495 \times 1.263 \times 100}$$

$$= 18.7 \text{ N/mm}^2 \text{ (tensile), At B.}$$

Example : 1.11

The supporting structure of a movable crane is shown in Fig. 1.16a. Determine the bending moment at section A-B, direct compressive load at A-B, maximum compressive stress and maximum shear stress.

Solution :

Re draw the critical section as shown in Fig. 1.16b

Inner radius $r_i = 380 \text{mm}$

Outer radius $r_o = 380 + 125 = 505 \text{ mm}.$

Radius of centroidal axis $r_c = 380 + \frac{125}{2} = 442.5 \text{ mm}$

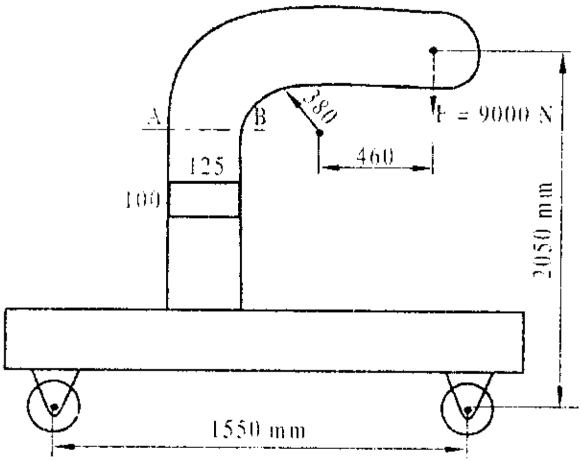


Fig. 1.16a

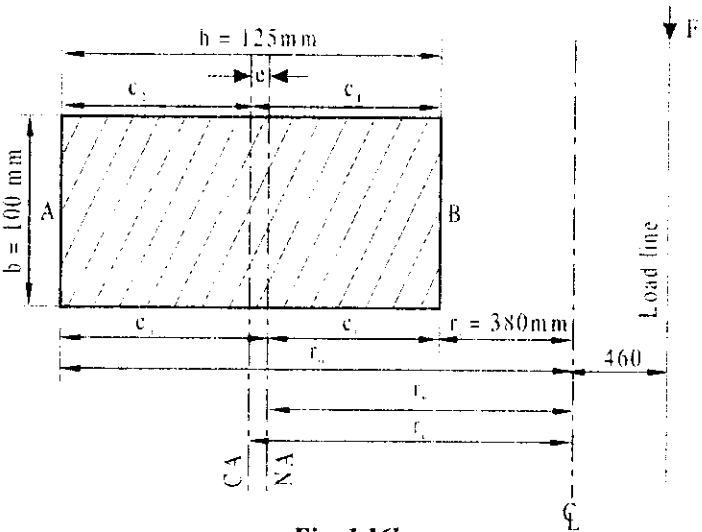


Fig. 1.16b

Radius of neutral axis $\quad r_n = \dfrac{h}{ln\left(\dfrac{r_o}{r_i}\right)}$ ----25.61

$$= \frac{125}{ln\left(\frac{505}{380}\right)} = 439.5416 \text{ mm}$$

$$e = r_c - r_n = 442.5 - 439.5416 = 2.9584 \text{ mm}$$

$$c_i = r_n - r_i = 439.5416 - 380 = 59.5416 \text{ mm}$$

$$c_o = r_o - r_n = 505 - 439.5416 = 65.4584 \text{ mm}$$

$$l = r_c + 460 = 442.5 + 460 = 902.5 \text{ mm}$$

$$A = bh = 100 \times 125 = 12500 \text{ mm}^2$$

Direct compressive load $F = 9000 \text{ N}$

Bending moment at section A-B $M_b = Fl = 9000 \times 902.5 = 8122500 \text{ Nmm}$

Direct **compressive** stress $\sigma_d = \frac{F}{A} = \frac{9000}{12500} = 0.72 \text{ N/mm}^2$

Bending stress at the inner fibre $\sigma_{bi} = -\frac{M_b c_i}{Aer_i} = -\frac{8122500 \times 59.5416}{12500 \times 2.9584 \times 380}$

$= -34.416 \text{ N/mm}^2$ (compressive)

Bending stress at the outer fibre $\sigma_{bo} = +\frac{M_b c_o}{Aer_o} = +\frac{8122500 \times 65.4584}{12500 \times 2.9584 \times 505}$

$= 28.47 \text{ N/mm}^2$ (tensile)

∴ Combined maximum stress at the inner fibre $\sigma_n = \sigma_d + \sigma_{bi} = -0.72 - 34.416$

$= 35.136 \text{ N/mm}^2$ (compressive)

Combined maximum stress at the outer fibre $\sigma_{ro} = \sigma_d + \sigma_{bo} = -0.72 + 28.47$

$= 27.75 \text{ N/mm}^2$ (tensile)

∴ Maximum compressive stress $= 35.136 \text{ N/mm}^2$ at B

Maximum shear stress $\tau_{max} = 0.5\sigma_{max} = 0.5 \times 35.136 = 17.568 \text{ N/mm}^2$ at B

The stress distribution at the critical section A - B is as shown in Fig. 1.16c.

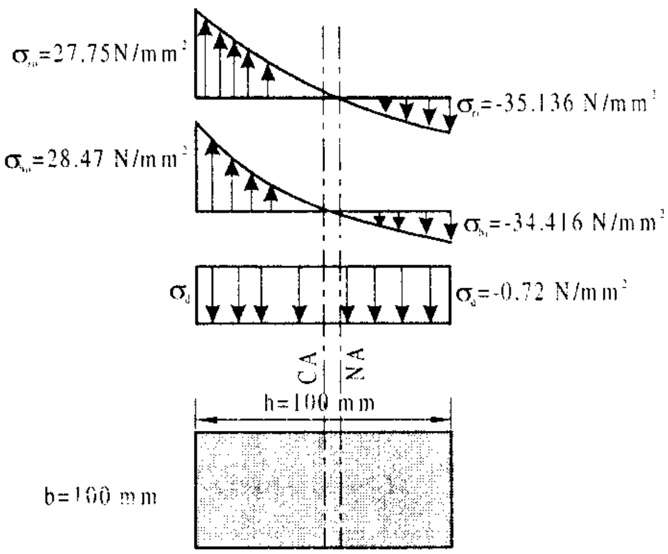


Fig. 1.16c

Example : 1.12

An offset bar is loaded as shown in Fig. 1.17a. What is the maximum offset distance 'x' if the allowable stress in tension is limited to 50 N/mm²

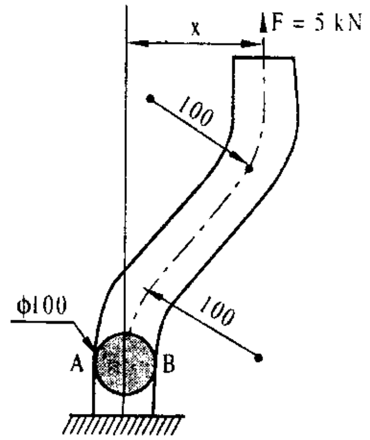


Fig. 1.17a

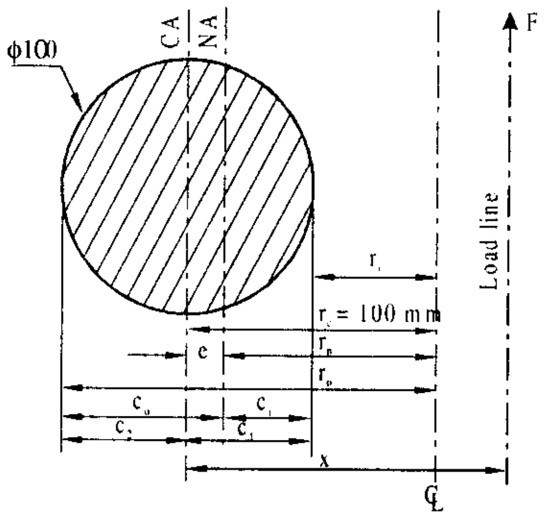


Fig. 1.17b

Solution :

Re draw the critical section as shown in Fig. 1.17b

Radius of centroidal axis $r_c = 100 \text{mm}$

Inner radius $r_i = 100 - \frac{100}{2} = 50 \text{ mm}$

Outer radius $r_o = 100 + \frac{100}{2} = 150 \text{ mm}$

Radius of neutral axis $r_n = \frac{\left(\sqrt{r_o}+\sqrt{r_i}\right)^2}{4} = \frac{\left(\sqrt{150}+\sqrt{50}\right)^2}{4} = 93.3 \text{ mm}$

$$e = r_c - r_n = 100 - 93.3 = 6.7 \text{ mm}$$

$$c_i = r_n - r_i = 93.3 - 50 = 43.3 \text{ mm}$$

$$c_o = r_o - r_n = 150 - 93.3 = 56.7 \text{ mm}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 100^2 = 7853.98 \text{ mm}^2$$

$$M_b = Fx = 5000x$$

Combined maximum tensile stress at the inner fibre

(i.e., at B) σ_n = Direct stress + bending stress

$$= \frac{F}{A} + \frac{M_b c_i}{A e r_i}$$

i.e.,

$$50 = \frac{5000}{7853.98} + \frac{(5000x)(43.3)}{7853.98 \times 6.7 \times 50}$$

$$\therefore x = 599.9 = \text{Maximum offset distance.}$$

Example : 1.13

The effect of two applied forces on the offset bar shown in Fig. 1.18a is pure couple which causes the same bending moment at every section of the beam. Determine maximum tension, compression and shear stress and state where each occurs.

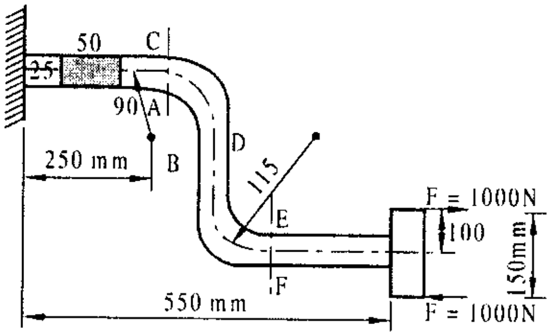


Fig. 1.18a

Solution :

The bending moment at every section is $150 \times 1000 = 150000$ Nmm $= M_b$

As the couple for equilibrium is as shown in Fig. 1.18a, tension occurs in the upper fibre CDE and compression occurs in the lower fibre ABF. Since the offset bar is subjected to pure couple, there is no direct stress.

(i) Consider the section A - C

Re draw the critical section at A - C as shown in Fig. 1.18b

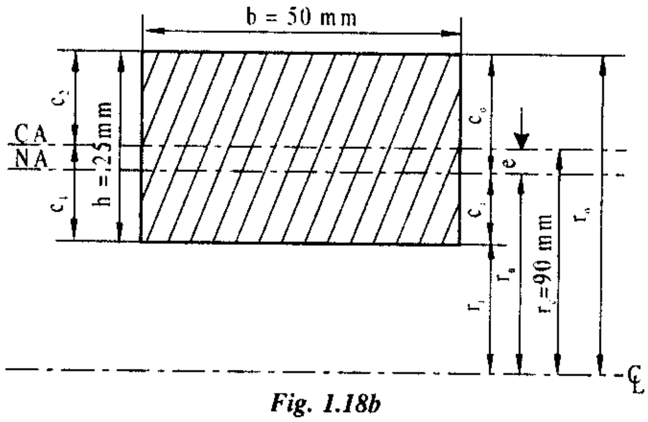


Fig. 1.18b

Radius of centroidal axis $r_c = 90\text{mm}$

Inner radius $r_i = 90 - \frac{25}{2} = 77.5 \text{ mm}$

Outer radius $r_o = 90 + \frac{25}{2} = 102.5 \text{ mm}$

Radius of neutral axis $r_n = \dfrac{h}{ln\left(\frac{r_o}{r_i}\right)}$ ---- 25.61

$= \dfrac{25}{ln\left(\frac{102.5}{77.5}\right)} = 89.418 \text{ mm}$

Distance of neutral axis from centroidal axis $e = r_c - r_n = 90 - 89.418 = 0.582 \text{ mm}$

Distance of neutral axis to inner fibre $c_i = r_n - r_i = 89.418 - 77.5 = 11.918 \text{ mm}$

Distance of neutral axis to outer fibre $c_o = r_o - r_n = 102.5 - 89.418 = 13.082 \text{ mm}$

Area of cross-section $A = bh = 50 \times 25 = 1250 \text{ mm}^2$

Stress at the inner fibre is compressive (at A) $= \dfrac{M_b c_i}{Aer_i} = \dfrac{150000 \times 11.918}{1250 \times 0.582 \times 77.5} = 31.707 \text{ N/mm}^2$

Stress at the outer fibre is tensile (at C) $= \dfrac{M_b c_o}{Aer_o} = \dfrac{150000 \times 13.082}{1250 \times 0.582 \times 102.5} = 26.315 \text{ N/mm}^2$

(ii) Consider the section E - F

Re draw the critical section at E - F as shown in Fig. 1.18c

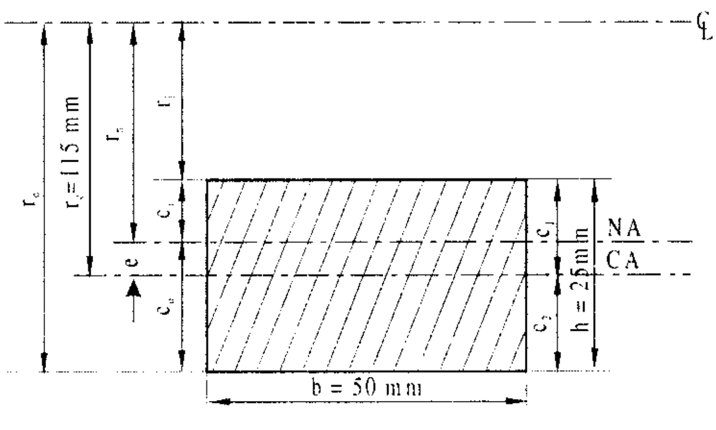


Fig. 1.18c

Radius of centroidal axis $r_c = 115\text{mm}$

Inner radius $r_i = 115 - \frac{25}{2} = 102.5 \text{ mm}$

Outer radius $r_o = 115+\frac{25}{2} = 127.5$ mm

Radius of neutral axis $r_n = \frac{h}{ln\left(\frac{r_o}{r_i}\right)} = \frac{25}{ln\left(\frac{127.5}{102.5}\right)} = 114.5457$ mm

Distance of neutral axis from centroidal axis $e = r_c - r_n = 115 - 114.5457 = 0.4543$ mm

Distance of neutral axis to inner fibre $c_i = r_n - r_i = 114.5457 - 102.5 = 12.0457$ mm

Distance of neutral axis to outer fibre $c_o = r_o - r_n = 127.5 - 114.5457 = 12.9543$ mm

Area of cross-section $A = bh = 50 \times 25 = 1250\ mm^2$

Stress at the inner fibre is tensile (at E) $= \frac{M_b c_i}{A e r_i} = \frac{150000 \times 12.0457}{1250 \times 0.4543 \times 102.5} = 31.042\ N/mm^2$

Stress at the outer fibre is compressive (at F) $= \frac{M_b c_o}{A e r_o} = \frac{150000 \times 12.9543}{1250 \times 0.4543 \times 127.5} = 26.837\ N/mm^2$

∴ Maximum tensile stress $= 31.042\ N/mm^2$ at E

Maximum compressive stress $= 31.707\ N/mm^2$ at A

Maximum shear stress $\tau_{max} = 0.5\,\sigma_{max} = 0.5 \times 31.707 = 15.8535\ N/mm^2$ at A

Example : 1.14

An open 'S' link is made from 25 mm diameter rod as shown in Fig. 1.19a. Determine the maximum tensile, compressive and shear stress

Solution :

(i) Consider the section P - Q

Redraw the critical section at P - Q as shown in Fig. 1.19b

Radius of centroidal axis $r_c = 100$ mm

Inner radius $r_i = 100 - \frac{25}{2} = 87.5$ mm

Outer radius $r_o = 100 + \frac{25}{2} = 112.5$ mm

Radius of neutral axis $r_n = \frac{\left(\sqrt{r_o} + \sqrt{r_i}\right)^2}{4}$ ----- 25.60

$= \frac{\left(\sqrt{112.5} + \sqrt{87.5}\right)^2}{4} = 99.6$ mm

Distance of neutral axis from centroidal axis $e = r_c - r_n = 100 - 99.6 = 0.4$ mm

Distance of neutral axis to inner fibre $c_i = r_n - r_i = 99.6 - 87.5 = 12.1$ mm

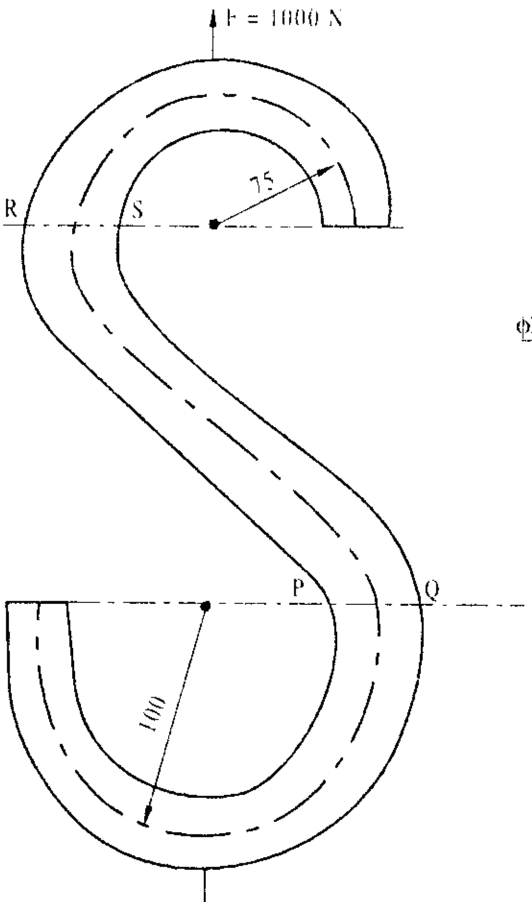


Fig. 1.19a

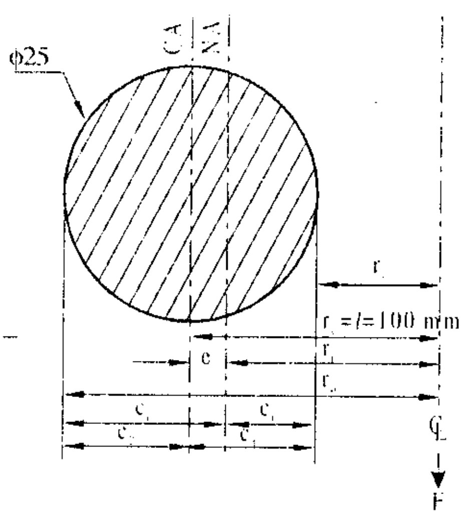


Fig. 1.19b

Distance of neutral axis to outer fibre $c_o = r_o - r_n = 112.5 - 99.6 = 12.9$ mm

Area of cross-section $A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 25^2 = 490.87$ mm^2

Distance from centroidal axis to force $l = r_c = 100$ mm

Bending moment about centroidal axis $M_b = F.l = 1000 \times 100 = 100000$ Nmm

Combined stress at the outer fibre (i.e., at Q) = Direct stress + bending stress

$$= \frac{F}{A} - \frac{M_b c_o}{A e r_o} = \frac{1000}{490.87} - \frac{100000 \times 12.9}{490.87 \times 0.4 \times 112.5}$$

$= -56.36$ N/mm^2 (compressive)

Combined stress at the inner fibre (i.e., at P) = Direct stress + bending stress

$$= \frac{F}{A} + \frac{M_b c_i}{A e r_i} = \frac{1000}{490.87} + \frac{100000 \times 12.1}{490.87 \times 0.4 \times 87.5}$$

$$= 72.466 \text{ N/mm}^2 \text{ (tensile)}$$

(ii) Consider the section R - S

Redraw the critical section at R - S as shown in Fig. 1.19c

$$r_c = 75\text{mm}$$

$$r_i = 75 - \frac{25}{2} = 62.5 \text{ mm}$$

$$r_o = 75 + \frac{25}{2} = 87.5 \text{ mm}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 25^2 = 490.87 \text{ mm}^2$$

$$r_n = \frac{\left(\sqrt{r_o} + \sqrt{r_i}\right)^2}{4} \text{ ---- } 25.60$$

$$= \frac{\left(\sqrt{87.5} + \sqrt{62.5}\right)^2}{4} = 74.4755 \text{ mm}$$

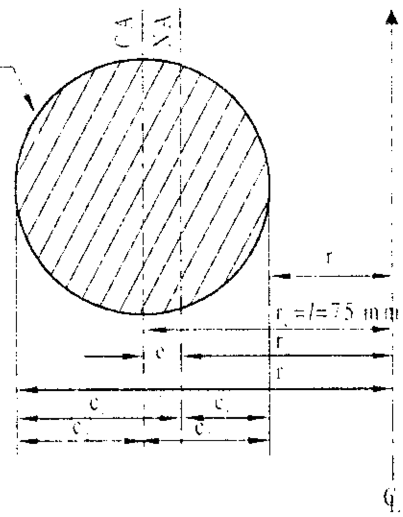


Fig. 1.19c

$$e = r_c - r_n = 75 - 74.4755 = 0.5245 \text{ mm}$$

$$c_i = r_n - r_i = 74.4755 - 62.5 = 11.9755 \text{ mm}$$

$$c_o = r_o - r_n = 87.5 - 74.4755 = 13.0245 \text{ mm}$$

$$l = r_c = 75 \text{ mm}$$

$$M_b = Fl = 1000 \times 75 = 75000 \text{ Nmm}$$

Combined stress at the outer fibre (at R) = Direct stress + Bending stress

$$= \frac{F}{A} - \frac{M_b c_o}{A e r_o} = \frac{1000}{490.87} - \frac{75000 \times 13.0245}{490.87 \times 0.5245 \times 87.5}$$

$$= -41.324 \text{ N/mm}^2 \text{ (compressive)}$$

Combined stress at the inner fibre (at S) = Direct stress + Bending stress

$$= \frac{F}{A} + \frac{M_b c_i}{A e r_i} = \frac{1000}{490.87} + \frac{75000 \times 11.9755}{490.87 \times 0.5245 \times 62.5}$$

$$= 55.816 \text{ N/mm}^2 \text{ (tensile)}$$

∴ Maximum tensile stress = 72.466 N/mm² at P

Maximum compressive stress = 56.36 N/mm² at Q

Maximum shear stress $\tau_{max} = 0.5\,\sigma_{max} = 0.5 \times 72.466 = 36.233$ N/mm² at P

Example : 1.15

Find the stresses at the inner and outer fibre of the machine frame of I section as shown in Fig. 1.20.

Solution :

$$r_i = 80 \text{ mm}; r_o = 80 + 120 = 200 \text{ mm}$$

Let AB be the reference line

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(10\times100)\left(\frac{10}{2}\right) + (10\times100)\left(10 + \frac{100}{2}\right) + (60\times10)\left(10 + 100 + \frac{10}{2}\right)}{(10\times100) + (10\times100) + (60\times10)}$$

$$= 51.538 \text{ mm}$$

$$r_c = r_i + \bar{x} = 80 + 51.538 = 131.538 \text{ mm}$$

$$A = a_1 + a_2 + a_3 = 10\times100 + 10\times100 + 60\times10 = 2600 \text{ mm}^2$$

$$r_n = \frac{A}{b_i ln\left(\frac{r_i + a_i}{r_i}\right) + b_2 ln\left(\frac{r_o - a_o}{r_i + a_i}\right) + b_o ln\left(\frac{r_o}{r_o - a_o}\right)} \quad \text{---- 25.63}$$

$$b_i = 100 \text{ mm}; a_i = 10 \text{ mm}; b_2 = 10 \text{ mm}; b_o = 60 \text{ mm}; a_o = 10 \text{ mm}$$

$$\therefore r_n = \frac{2600}{100ln\left(\frac{80+10}{80}\right) + 10ln\left(\frac{200-10}{80+10}\right) + 60ln\left(\frac{200}{200-10}\right)} = 116.445 \text{ mm}$$

$$e = r_c - r_n = 131.538 - 116.455 = 15.093 \text{ mm}$$

$$c_i = r_n - r_i = 116.455 - 80 = 36.445 \text{ mm}$$

$$c_o = r_o - r_n = 200 - 116.455 = 83.555 \text{ mm}$$

$$l = r_c = 131.538 \text{ mm}$$

$$M_b = Fl = 20{,}000 \times 131.538 = 2630760 \text{ Nmm}$$

Combined maximum stress at the inner fibre

$$\sigma_{ri} = \text{Direct stress + Bending stress}$$

$$= \frac{F}{A} + \frac{M_b c_i}{Aer_i} = \frac{20{,}000}{2600} + \frac{2630760\times36.445}{2600\times15.093\times80}$$

$$= 38.23 \text{ N/mm}^2 \text{ (tensile)}$$

Combined maximum stress at the outer fibre

$$\sigma_{ro} = \text{Direct stress + Bending stress}$$

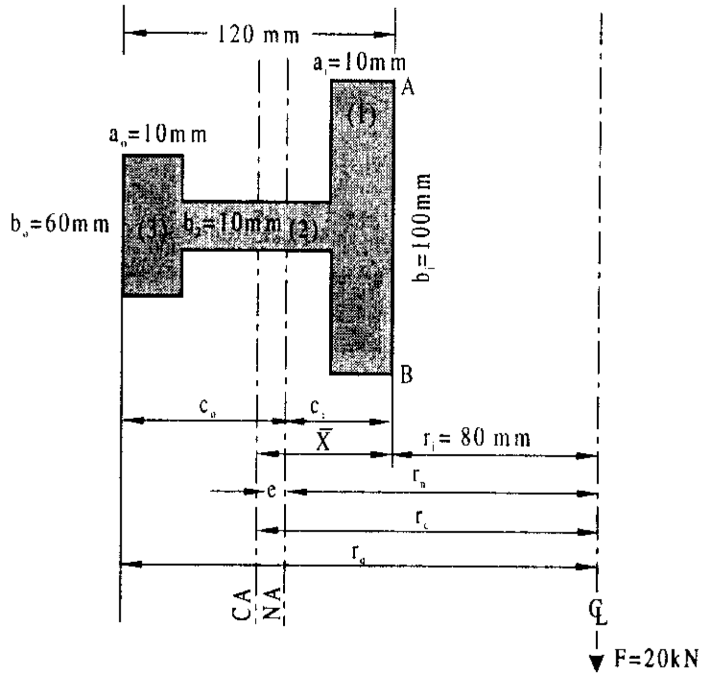


Fig. 1.20

$$= \frac{F}{A} - \frac{M_b c_o}{Aer_o} = \frac{20000}{2600} - \frac{2630760 \times 83.555}{2600 \times 15.093 \times 200}$$

$= -20.31$ N/mm² (compressive)

Maximum shear stress $\tau_{max} = 0.5\,\sigma_{max} = 0.5 \times 38.23 = 19.115$ N/mm² at the inner fibre.

Example : 1.16

Determine the capacity of a frame of a punch press shown in Fig. 1.21a. The material from which the frame is made has an allowable stress of 80 N/mm² along the section A - B.

Solution :

Redraw the critical section as shown in Fig. 1.21b

Inner radius $r_i = 60$ mm

Outer radius $r_o = 60 + 20 + 40 = 120$ mm

Let AB be the reference line

$$\therefore \bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(50 \times 20)\left(\frac{20}{2}\right) + (40 \times 20)\left(20 + \frac{40}{2}\right)}{(50 \times 20) + (40 \times 20)}$$

$= 23.33$ mm

Radius of centroidal axis $r_n = r_i + \bar{x} = 60 + 23.33 = 83.33$ mm

Area of cross section $A = a_1 + a_2 = 50 \times 20 + 40 \times 20 = 1800$ mm²

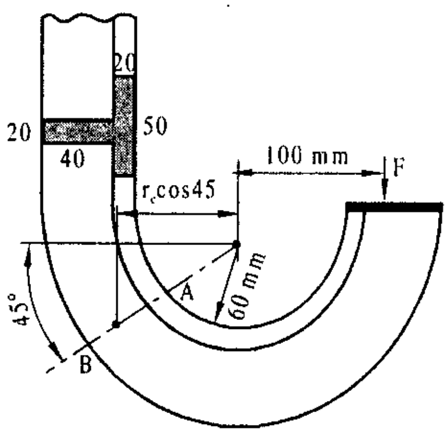


Fig. 1.21a

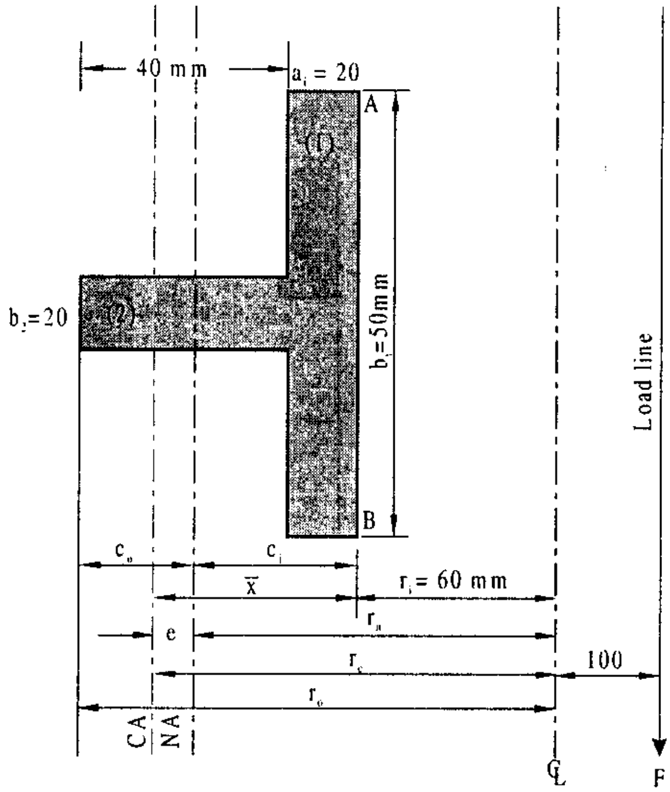


Fig. 1.21b

- Radius of neutral axis $r_n = \dfrac{A}{b_i ln\left(\dfrac{r_i + a_i}{r_i}\right) + b_2 ln\left(\dfrac{r_o - a_o}{r_i + a_i}\right) + b_o ln\left(\dfrac{r_o}{r_o - a_o}\right)}$ ----25.63

$b_i = 50$ mm ; $a_i = 20$ mm ; $b_2 = 20$ mm ; $b_o = 0$; $a_o = 0$

$$\therefore \; r_n = \frac{1800}{50ln\left(\dfrac{60+20}{60}\right) + 20ln\left(\dfrac{120-0}{60+20}\right) + 0} = 80.02 \text{ mm}$$

Distance of neutral axis from centroidal axis $e = r_c - r_n = 83.33 - 80.02 = 3.31$ mm

Distance of inner radius from neutral axis $c_i = r_n - r_i = 80.02 - 60 = 20.02$ mm

Distance of outer radius from neutral axis $c_o = r_o - r_n = 120 - 80.02 = 39.98$ mm

Distance from centroidal axis to force $l = r_c \cos 45 + 100$

$= 83.33 \cos 45 + 100 = 158.923$ mm

Bending moment about centroidal axis $M_b = F.l = 158.923$ F

$$\text{Direct load } F_d = F\cos 45$$

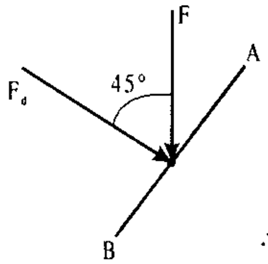
Combined stress at the inner fibre σ_{ri} = Direct stress + Bending stress

$$= \frac{F_d}{A} + \frac{M_b c_i}{A e r_i}$$

$$\text{i.e., } 80 = \frac{F\cos 45}{1800} + \frac{158.923F \times 20.02}{1800 \times 3.31 \times 60}$$

$$\therefore F = 8608.6\text{N}$$

Combined stress at the outer fibre σ_{ro} = Direct stress + Bending stress



$$= \frac{F_d}{A} - \frac{M_b c_o}{A e r_o}$$

$$\text{i.e., } 80 = \frac{F\cos 45}{1800} - \frac{(158.923F)\times(39.98)}{1800 \times 3.31 \times 120}$$

$$\therefore F = 9418.4\text{N}$$

$\therefore$ Capacity of the frame F = 8608.6N [Select the smaller value].

Example : 1.17

The frame of a punch press has a shape as shown in Fig. 1.22a. The dimensions are marked in mm. Analysis along the section A - B and determine the maximum stress developed at points A and B taking curvature into effect.

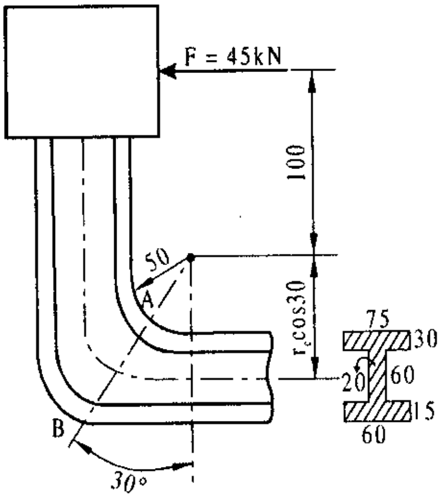


Fig. 1.22a

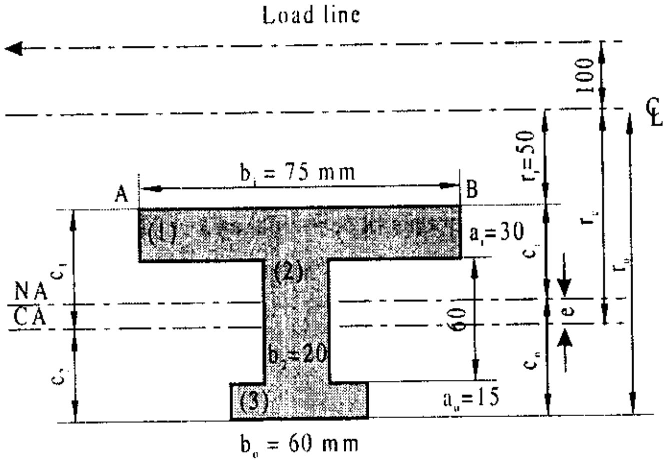


Fig. 1.22b

Solution :

Re draw the critical section as shown in Fig. 1.22b

Inner radius r_i = 50 mm

Outer radius r_o = 50 + 30 + 60 + 15 = 155 mm

Let AB be the reference line

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(75\times30)\left(\frac{30}{2}\right) + (60\times20)\left(30+\frac{60}{2}\right) + (60\times15)\left(30+60+\frac{15}{2}\right)}{(75\times30)+(60\times20)+(60\times15)}$$

$= 44.482\,\text{Nmm}$

Radius of centroidal axis $r_c = r_i + \bar{x} = 50 + 44.482 = 94.482\,\text{mm}$

Area of cross section $A = a_1 + a_2 + a_3 = (75\times30) + (60\times20) + (60\times15) = 4350\,\text{mm}^2$

Radius of neutral axis

$$r_n = \frac{A}{b_i ln\left(\frac{r_i + a_i}{r_i}\right) + b_2 ln\left(\frac{r_o - a_o}{r_i + a_i}\right) + b_o ln\left(\frac{r_o}{r_o - a_o}\right)} \quad \text{---- 25.63}$$

$b_i = 75$ mm; $a_i = 30$ mm; $b_2 = 20$ mm; $b_o = 60$ mm; $a_o = 15$ mm

$$\therefore\ r_n = \frac{4350}{75ln\left(\frac{30+50}{50}\right) + 20ln\left(\frac{155-15}{50+30}\right) + 60ln\left(\frac{155}{155-15}\right)} = 82.78\text{ mm}$$

Distance of neutral axis from centroidal axis $e = r_c - r_n = 94.48 - 82.78 = 11.7$ mm

Distance of inner radius from neutral axis $c_i = r_n - r_i = 82.78 - 50 = 32.78$ mm

Distance of outer radius from neutral axis $c_o = r_o - r_n = 155 - 82.78 = 72.22$ mm

Distance from centroidal axis to force $l = r_c \cos 30 + 100$

$= 94.482 \cos 30 + 100 = 181.82$ mm

Direct load $F_d = F\cos30 = 45{,}000 \cos30 = 38971.1$N

Bending moment about centroidal axis $M_b = Fl = 45{,}000 \times 181.82 = 8181500$ Nmm

Combined maximum stress at the inner fibre σ_{ri} = Direct stress + Bending stress

$$= \frac{F_d}{A} + \frac{M_b c_i}{Aer_i} = \frac{38971.1}{4350} + \frac{8181500\times32.78}{4350\times11.7\times50}$$

$= 114.25$ N/mm^2 (tensile)

Combined maximum stress at the outer fibre σ_{ro} = Direct stress + Bending stress

$$= \frac{F_d}{A} - \frac{M_b c_o}{Aer_o} = \frac{38971.1}{4350} - \frac{8181500\times72.22}{4350\times11.7\times155}$$

$= -65.9$ N/mm^2 (compressive)

Maximum shear stress $\tau_{max} = 0.5 \times 114.25 = 57.125$ N/mm^2 at the inner fibre.

Example : 1.18

For the section A - B shown in Fig. 1.23 a find bending moment, direct tensile force, maximum tensile stress, maximum compressive stress, maximum shear stress, and their locations.

(B.U. Aug/Sep. 2001)

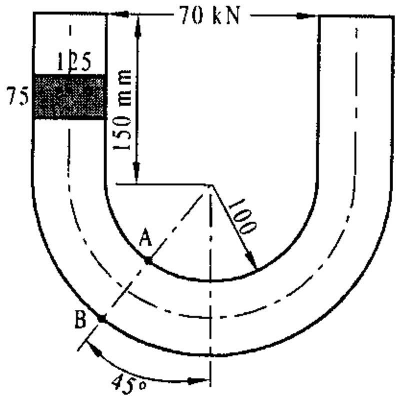


Fig. 1.23a

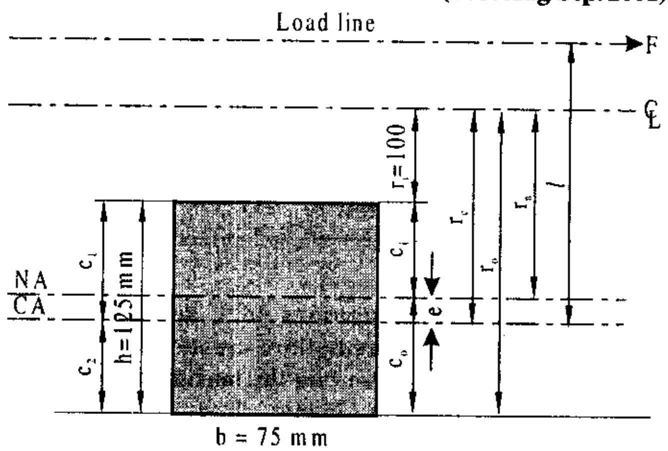


Fig. 1.23b

Solution :

Re draw the critical section as shown in Fig. 1.23b

Inner radius $r_i = 100\text{mm}$

Outer radius $r_o = 100 + 125 = 225\text{ mm}$

Radius of centroidal axis $r_c = 100 + \frac{125}{2} = 162.5\text{ mm}$

Radius of neutral axis $r_n = \frac{h}{ln\left(\frac{r_o}{r_i}\right)} = \frac{125}{ln\left(\frac{225}{100}\right)} = 154.14\text{ mm}$

$$e = r_c - r_n = 162.5 - 154.14 = 8.36\text{ mm}$$

$$c_i = r_n - r_i = 154.14 - 100 = 54.14\text{ mm}$$

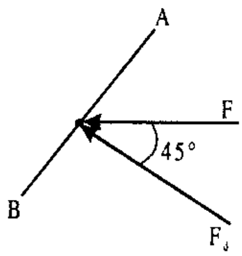
$$c_o = r_o - r_n = 225 - 154.14 = 70.86\text{ mm}$$

$$l = r_c \cos 45 + 150 = 162.5 \cos 45 + 150 = 264.9\text{ mm}$$

$$M_b = Fl = 70{,}000 \times 264.9 = 18543000\text{ Nmm}$$

$$A = bh = 75 \times 125 = 9375\text{ mm}^2$$

Direct tensile force $F_d = F\cos 45 = 70{,}000 \cos 45 = 49497.47\text{ N}$



Maximum tensile stress at the inner fibre

$$\sigma_{ri} = \text{Direct stress} + \text{Bending stress}$$

$$= \frac{F_d}{A} + \frac{M_b c_i}{Aer_i} = \frac{49497.47}{9375} + \frac{18543000 \times 54.14}{9375 \times 8.36 \times 100}$$

$= 133.37$ N/mm² at A

Maximum compressive stress at the outer fibre

σ_m = Direct stress + Bending stress

$$= \frac{F_d}{A} - \frac{M_b c_o}{Aer_o} = \frac{49497.47}{9375} - \frac{18543000 \times 70.86}{9375 \times 8.36 \times 225}$$

$= -69.2$ N/mm² at B

Maximum shear stress $\tau_{max} = 0.5\,\sigma_{max} = 0.5 \times 133.37 = 66.685$ N/mm² at A.

Example : 1.19

Determine the dimensions of 'I' section shown in Fig. 1.24 in which both extreme fibre stresses are numerically equal in pure bending. Also find the maximum stress if the load is 20 kN and $b_i + b_o = 125$ mm

Solution :

Redraw the critical section as shown in Fig. 1.24

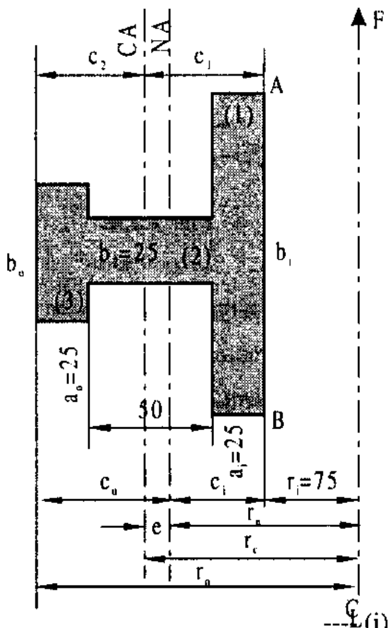


Fig. 1.24

Inner radius $r_i = 75$ mm

Outer radius $r_o = 75 + 25 + 50 + 25 = 175$ mm

Since it is subjected to pure bending and the bending stresses at the extreme fibres are numerically equal.

$$\frac{M_b c_i}{Aer_i} = \frac{M_b c_o}{Aer_o}$$

$$\text{i.e., } c_i = c_o \frac{r_i}{r_o} = c_o \left(\frac{75}{175}\right)$$

$$\therefore\ c_i = 0.4285\, c_o \qquad \text{----(i)}$$

From Fig. 1.24,

$$c_i + c_o = 25 + 50 + 25 = 100 \qquad \text{----(ii)}$$

Substituting (i) in equation (ii)

$$0.4285\, c_o + c_o = 100$$

$$\therefore\ c_o = 70 \text{ mm and } c_i = 30 \text{mm.}$$

Area of cross-section $A = 25b_i + 50 \times 25 + 25b_o = 25\,(b_i + b_o) + 1250$

$= 25 \times 125 + 1250 = 4375$ mm²

Distance of inner radius from neutral axis $c_i = r_n - r_i$

i.e., $30 = r_n - 75$

$\therefore\ r_n = 105$ mm = Radius of neutral axis

$$\therefore r_n = \frac{A}{b_i ln\left(\frac{r_i + a_i}{r_i}\right) + b_2 ln\left(\frac{r_o - a_o}{r_i + a_i}\right) + b_o ln\left(\frac{r_o}{r_o - a_o}\right)} \qquad \text{----25.63}$$

$$\text{i.e., } 105 = \frac{4375}{b_i ln\left(\frac{75+25}{75}\right) + 25 ln\left(\frac{175-25}{75+25}\right) + b_o ln\left(\frac{175}{175-25}\right)}$$

$$= \frac{4375}{0.2876 b_i + 10.136 + 0.154 b_o}$$

i.e., $30.198\, b_i + 1064.28 + 16.17\, b_o = 4375$

i.e., $30.198\, b_i + 16.17\,(125 - b_i) = 3310.72$ $\qquad (\because b_i + b_o = 125)$

$\therefore$ **b_i = 91.92 mm**

$\therefore$ **b_o = 33.08 mm**

Let AB be the reference line

$$\therefore \bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

$$= \frac{91.92 \times 25 \times \frac{25}{2} + (50 \times 25)\left(25 + \frac{50}{2}\right) + (33.08 \times 25)\left(25 + 50 + \frac{25}{2}\right)}{4375}$$

$= 37.391$ mm

$\therefore r_c = r_i + \bar{x} = 75 + 37.391 = 112.391 \text{ mm} = l$

$\therefore$ Bending moment about centroidal axis $M_b = Fl = 20{,}000 \times 112.391 = 2247820$ Nmm

Distance of neutral axis from centroidal axis $e = r_c - r_n = 112.391 - 105 = 7.391$ mm

Maximum stress at the inner fibre $\qquad \sigma_{ri} = \dfrac{M_b c_i}{A e r_i}$

$$= \frac{2247820 \times 30}{4375 \times 7.391 \times 75} = 27.806 \text{ N/mm}^2 \text{ (tensile)}$$

Maximum stress at the outer fibre $\qquad \sigma_{ro} = 27.806$ N/mm^2 (compressive).

Example : 1.20

A curved beam with a circular centre line has trapezoidal cross-section and is subjected to pure bending in its plane of symmetry. The concave side face is 100 mm and 100 mm from the centre line of curvature. The depth is 100 mm. Find the proper value of other parallel face, if the maximum extreme fibre stresses in tension and compression are numerically equal. Determine the magnitude of couple if it is subjected to a maximum fibre stress of 50 N/mm^2.

Solution :

Inner radius $\qquad r_i = 100$ mm

Outer radius $r_o = 100 + 100 + 200\text{ mm}$

Since it is subjected to pure bending and the bending stresses at the extreme fibres are numerically equal.

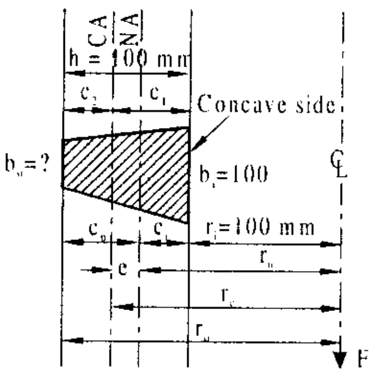


Fig. 1.25

$$\frac{M_b c_i}{A e r_i} = \frac{M_b c_o}{A e r_o}$$

$$\text{i.e., } c_i = c_o\left(\frac{r_i}{r_o}\right) = c_o\left(\frac{100}{200}\right) = 0.5\,c_o$$

From Fig. 1.25

$$c_i + c_o = 100$$

$$\text{i.e., } 0.5\,c_o + c_o = 100$$

$$\therefore c_o = 66.667\text{ mm and } c_i = 33.333\text{ mm}$$

Distance of inner radius from neutral axis $c_i = r_n - r_i$

$$\text{i.e., } 33.333 = r_n - 100$$

$$\therefore r_n = 133.333\text{ mm}$$

$$\text{Also } r_n = \frac{\frac{1}{2}h(b_i + b_o)}{\left(\frac{b_i r_o - b_o r_i}{h}\right) ln\left(\frac{r_o}{r_i}\right) - (b_i - b_o)} \qquad \text{----25.62}$$

$$\text{i.e., } 133.333 = \frac{\frac{1}{2} \times 100(100 + b_o)}{\left(\frac{100 \times 200 - b_o \times 100}{100}\right) ln\left(\frac{200}{100}\right) - (100 - b_o)}$$

$$= \frac{5000 + 50b_o}{138.629 - 0.693b_o - 100 + b_o}$$

$$\therefore \mathbf{b_o = 16.54\ mm}$$

Distance of centroidal axis from inner fibre $c_i = \frac{h}{3}\left(\frac{b_i + 2b_o}{b_i + b_o}\right)$

$$= \frac{100}{3}\left(\frac{100 + 2 \times 16.54}{100 + 16.54}\right) = 38.064\text{ mm}$$

$\therefore$ Radius of centroidal axis $r_c = r_i + c_i = 100 + 38.064 = 138.064\text{ mm}$

Distance of neutral axis from centroidal axis $e = r_c - r_n = 138.064 - 133.333 = 4.731\text{ mm}$

Area of cross-section $A = \frac{1}{2}h(b_i + b_o) = \frac{1}{2}\,100\,[100 + 16.54] = 5827\text{ mm}^2$

$$\text{Maximum stress} = \frac{M_b c_i}{A e r_i}$$

[Since it is subjected to pure bending, there is no direct stress]

$$\text{i.e., } 50 = \frac{M_b \times 33.333}{5827 \times 4.731 \times 100}$$

$$\therefore M_b = 4135171.9 \text{Nmm.}$$

Example : 1.21

Determine the safe load F that the frame of a punch press shown in Fig. 1.26a can carry considering the cross section along A - A for an allowable tensile stress of 100 MPa. What is the stress at the outer fibre for the above load? What will be the stress at the inner fibre, if the beam is treated as straight beam for the above load.

(V.T.U. July/Aug. 2004)

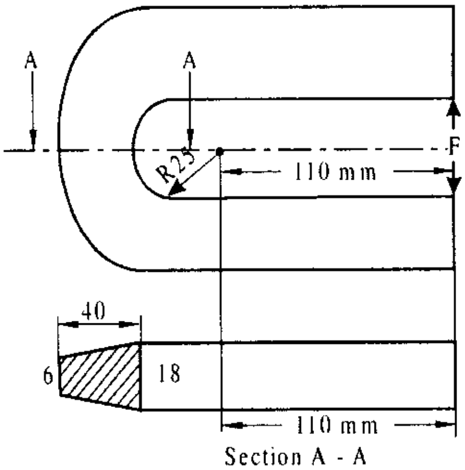


Fig. 1.26a

Solution :

(a) Beam is treated as curved beam

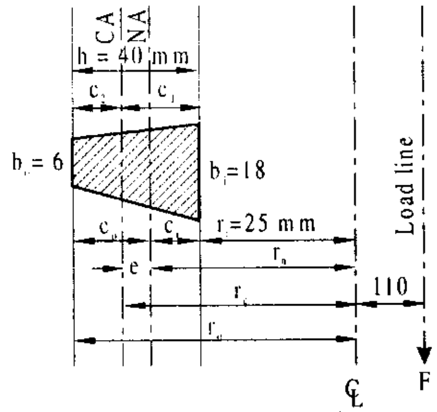


Fig. 1.26b

Redraw the critical section as shown in Fig. 1.26b

Inner radius $r_i = 25\text{mm}$

Outer radius $r_o = 25 + 40 = 65 \text{ mm}$

Distance of centroidal axis from the inner fibre c_i

$$= \frac{h}{3}\left(\frac{b_i + 2b_o}{b_i + b_o}\right) = \frac{40}{3}\left(\frac{18 + 2 \times 6}{18 + b}\right) = 16.667 \text{ mm}$$

$\therefore$ Radius of centroidal axis $r_c = r_i + c_i = 25 + 16.667 = 41.667$ mm

Radius of neutral axis $$r_n = \frac{\frac{1}{2}h(b_i + b_o)}{\left(\frac{b_i r_o - b_o r_i}{h}\right) ln\left(\frac{r_o}{r_i}\right) - (b_i - b_o)}$$

$$= \frac{\frac{1}{2} \times 40(18+6)}{\left(\frac{18 \times 65 - 6 \times 25}{40}\right) ln\left(\frac{65}{25}\right) - (18-6)} = 38.818 \text{ mm}$$

$$e = r_c - r_n = 41.667 - 38.818 = 2.849 \text{ mm}$$

$$c_i = r_n - r_i = 38.818 - 25 = 13.818 \text{ mm}$$

$$c_o = r_o - r_n = 65 - 38.818 = 26.182 \text{ mm}$$

$$A = \frac{1}{2} h (b_i + b_o) = \frac{1}{2} \times 40 [18 + 6] = 480 \text{ mm}^2$$

$$l = 110 + r_c = 110 + 41.667 = 151.667 \text{ mm}$$

$$M_b = Fl = 151.667 \text{ F}$$

Combined maximum tensile stress at the inner fibre

$$\sigma_{ri} = \text{Direct stress + Bending stress}$$

$$= F_d + \sigma_{bi} = \frac{F}{A} + \frac{M_b c_i}{Aer_i}$$

$$\text{i.e., } 100 = \frac{F}{480} + \frac{(151.667F)(13.818)}{(480)(2.849)(25)}$$

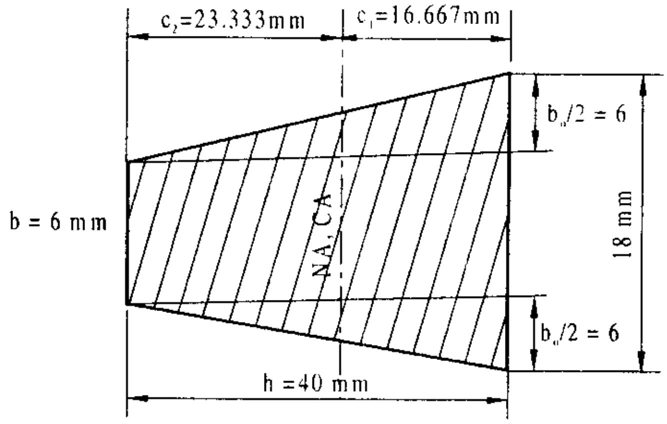
$$\therefore F = 1577.7\text{N = safe load.}$$

Maximum stress at the outer fibre σ_{ro} = Direct stress + Bending stress

$$= \frac{F}{A} - \frac{M_b c_o}{Aer_o} = \frac{1577.7}{480} - \frac{(151.667 \times 1577.7)(26.182)}{480 \times 2.849 \times 65}$$

$$= -67.194 \text{ N/mm}^2 \text{ (compressive)}$$

(b) Beam is treated as straight beam



$$b = 6\text{mm}$$

$$b_o = 18 - 6 = 12 \text{ mm}$$

$$c_1 = 16.667 \text{ mm}$$

$$c_2 = 40 - 16.667 = 23.333 \text{ mm}$$

$$A = 480 \text{ mm}^2$$

$$M_b = 151.667 \times 1577.7$$

$$= 239285 \text{ Nmm}$$

Fig. 1.26c

From Table 2.6 (Old DDHB Vol. I); Table 2.7 (New DDHB Vol. I)

Moment of Inertia $I = \dfrac{\left(6b^2 + 6bb_o + b_o^2\right)}{36(2b + b_o)} h^3$

$= \dfrac{\left(6 \times 6^2 + 6 \times 6 \times 12 + 12^2\right)40^3}{36[2 \times 6 + 12]} = 58666.667 \text{ mm}^4$

Combined stress at the inner fibre $\sigma_{ri} = \sigma_d + \sigma_{bi} = \dfrac{F}{A} + \dfrac{M_b c_i}{I}$

$= \dfrac{1577.7}{480} + \dfrac{239285 \times 16.667}{58666.667} = 71.267 \text{ N/mm}^2 \text{ (tensile)}$

Example : 1.22

The horizontal cross section of a crane hook is an isosceles triangle of 120 mm deep, the inner width being 90 mm. The hook carries a load of 50 kN. Inner radius of curvature is 100 mm. The load line passes through the centre line of curvature. Determine the stresses at the extreme fibres. VTU, June/July 2009

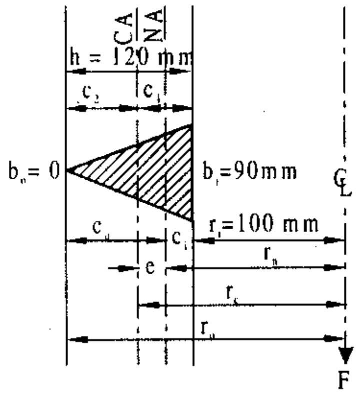


Fig. 1.27

Solution :

Draw the critical section as shown in Fig. 1.27

Inner radius $r_i = 100$ mm

Outer radius $r_o = 100 + 120 + 220$ mm.

Distance of centroidal axis from inner fibre

$c_i = \dfrac{h}{3} = \dfrac{120}{3} = 40 \text{ mm}$

∴ Radius of centroidal axis $r_c = r_i + c_i = 100 + 40 = 140$ mm

Area of cross-section $= \dfrac{1}{2} b_i h = \dfrac{1}{2} \times 90 \times 120 = 5400 \text{ mm}^2$

Radius neutral axis $r_n = \dfrac{\frac{1}{2} h(b_i + b_o)}{\left(\dfrac{b_i r_o - b_o r_i}{h}\right) ln\left(\dfrac{r_o}{r_i}\right) - (b_i - b_o)}$ ---- 25.62

$b_i = 90 \text{ mm} ; b_o = 0$

$\therefore r_n = \dfrac{5400}{\left(\dfrac{90 \times 220}{120}\right) ln\left(\dfrac{220}{100}\right) - 90} = 134.68 \text{ mm}$

$e = r_c - r_n = 140 - 134.68 = 5.32$ mm

$c_i = r_n - r_i = 134.68 - 100 = 34.68$ mm

$$c_o = r_o - r_n = 220 - 134.68 = 85.32 \text{ mm}$$
$$l = r_c = 140 \text{ mm}$$
$$M_b = Fl = 50000 \times 140 = 7 \times 10^6 \text{ Nmm}$$

$\therefore$ Combined maximum stress at the inner fibre σ_{ri} = Direct stress + Bending stress = $\frac{F}{A} + \frac{M_b c_i}{A e r_i}$

$$= \frac{50{,}000}{5400} + \frac{7 \times 10^6 \times 34.68}{5400 \times 5.32 \times 100} = 93.762 \text{ N/mm}^2 \text{ (tensile)}$$

Combined maximum stress at the outer fibre σ_{ro} = Direct stress + Bending stress = $\frac{F}{A} - \frac{M_b c_o}{A e r_o}$

$$= \frac{50{,}000}{5400} - \frac{7 \times 10^6 \times 85.32}{5400 \times 5.32 \times 220} = -85.238 \text{ N/mm}^2 \text{ (compressive)}$$

Maximum shear stress $\tau_{max} = 0.5\,\sigma_{max} = 0.5 \times 93.762 = 46.881$ N/mm², at the inner fibre.

Stress in Closed Ring

A closed ring is an example of a curved beam with restrained ends. Section A - A is the section at which the load is acting. Section B - B is 90° away from the point of application of load. Let the ring be subjected to a central load F as shown is Fig. 1.28. The bending moment at any cross section of the ring is given by $\frac{Fr}{2}\left(\cos\theta - \frac{2}{\pi}\right)$

$$\text{At section A - A, } \theta = \frac{\pi}{2} = 90°$$

$\therefore$ Bending moment at A - A, $M_{b_A} = -0.318$ Fr ---- 25.68

negative sign refers to tensile load

At section B - B, $\theta = 0$

$\therefore$ Bending moment at B - B, $M_{b_B} = +0.182$ Fr ---- 25.69

positive sign refers to tensile load.

Direct stress at any cross-section DD at an angle θ with horizontal

$$\sigma_d = \frac{F\cos\theta}{2A} \quad \text{---- 25.72}$$

The general expression for bending moment at any cross-section DD at an angle θ with the horizontal $M = M_{b_B} - \frac{1}{2}\,Fr\,(1 - \cos\theta)$ ---- 25.71

At section A - A

Bending moment $M_{b_A} = \mp 0.318$ Fr ---- 25.68

Where $r = r_c$, negative sign refers to tensile load and positive sign refers to compressive load

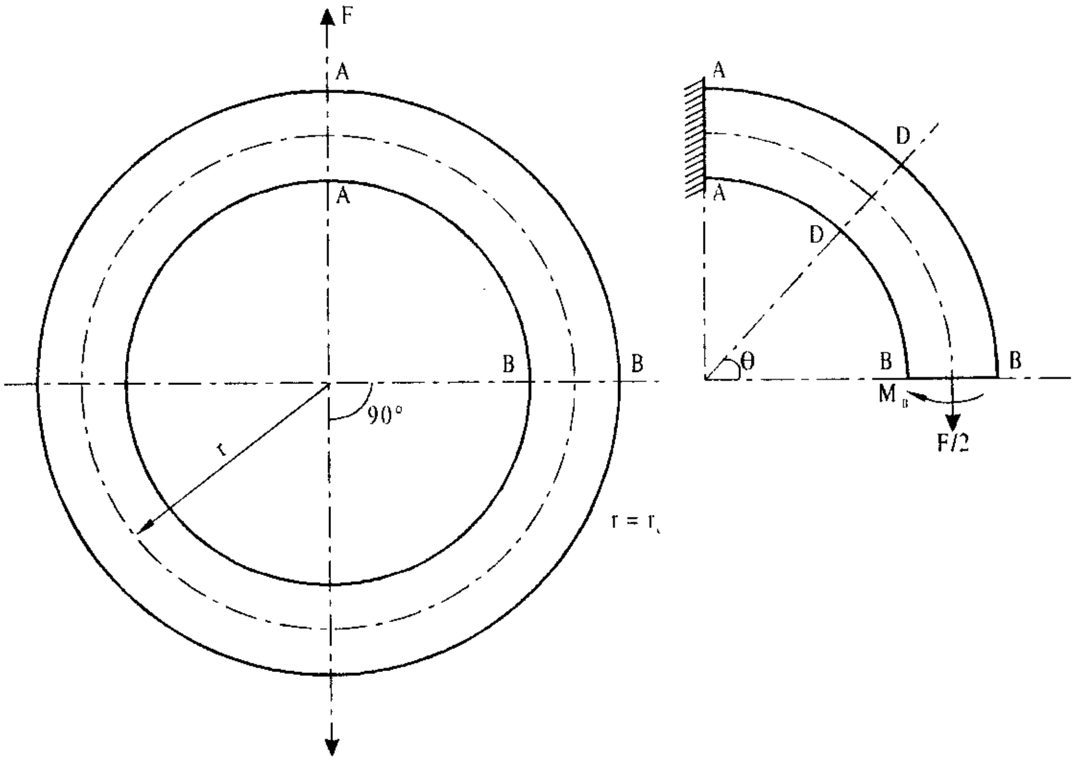


Fig. 1.28 [25.13 DDHB]

Maximum stress at the inner fibre $\sigma_{r_{i_A}} = \pm \dfrac{M_{b_A} c_i}{A e r_i}$

Maximum stress at the fibre $\sigma_{r_{o_A}} = \pm \dfrac{M_{b_A} c_o}{A e r_o}$

At section B - B

Bending moment $M_{b_B} = \pm 0.182\, Fr$ ---- 25.69

Where $r = r_c$, positive sign refers tensile load and negative sign refers to compressive load

Maximum stress at the inner fibre $\sigma_{r_{i_B}} = \dfrac{F}{2A} \pm \dfrac{M_{b_B} c_i}{A e r_i}$

Maximum stress at the outer fibre $\sigma_{r_{o_B}} = \dfrac{F}{2A} \pm \dfrac{M_{b_B} c_o}{A e r_o}$

Example : 1.23

Determine the stresses induced in a circular ring of circular cross-section of 25 mm diameter subjected to a tensile load of 6500N. The inner diameter of the ring is 60 mm.

Solution :

The circular ring and its critical section are as shown in Fig. 1.29a and 1.29b respectively.

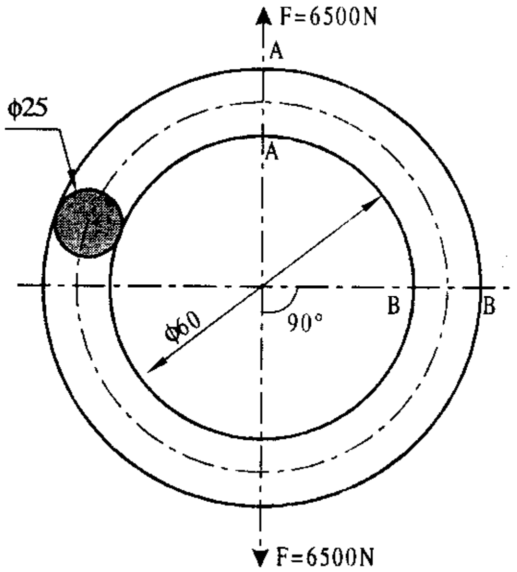


Fig. 1.29a

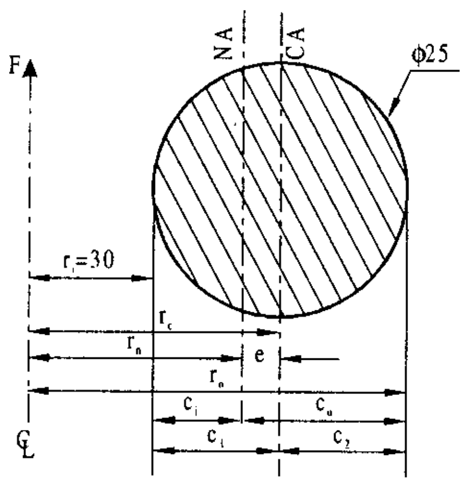


Fig. 1.29b

Inner radius $r_i = \frac{60}{2} = 30\,\text{mm}$

Outer radius $r_o = 30 + 25 = 55\,\text{mm}$

Radius of centroidal axis $r_c = 30 + \frac{25}{2} = 42.5\,\text{mm}$

Radius neutral axis $r_n = \frac{\left(\sqrt{r_o} + \sqrt{r_i}\right)^2}{4}$

$$= \frac{\left(\sqrt{55} + \sqrt{30}\right)^2}{4} = 41.56\,\text{mm}$$

Distance from neutral axis to centroidal axis $e = r_c - r_n = 42.5 - 41.56 = 0.94$ mm

Distance of inner radius from neutral axis $c_i = r_n - r_i = 41.56 - 30 = 11.56$ mm

Distance of outer radius from neutral axis $c_o = r_o - r_n = 55 - 41.56 = 13.44$ mm

Direct stress at any cross section at an angle θ with horizontal

$$\sigma_d = \frac{F\cos\theta}{2A}$$

Consider the section A - A

At section A - A, θ = 90° with respect to horizontal

$$\therefore \text{ Direct stress} \quad \sigma_d = \frac{F\cos 90}{2A} = 0$$

$$\text{Bending moment} \quad M_{b_A} = -0.318\ Fr \qquad \text{---- 25.68}$$

Where $r = r_c$, negative sign refers to tensile load

$$\therefore M_{b_A} = -0.318 \times 6500 \times 42.5 = -87847.5 \text{ Nmm}$$

This couple produces compressive stress at the inner fibre and tensile stress at the outer fibre

$$\therefore \text{ Maximum stress at the inner fibre } \sigma_{ri_A} = \text{Direct stress + Bending stress} = 0 - \frac{M_{b_A} c_i}{Aer_i}$$

$$= 0 - \frac{87847.5 \times 11.56}{490.874 \times 0.94 \times 30} = -73.36 \text{ N/mm}^2 \text{ (Compressive)}$$

$$\text{Maximum stress at the outer fibre } \sigma_{ro_A} = \text{Direct stress + Bending stress} = 0 + \frac{M_{b_A} c_o}{Aer_o}$$

$$= +\frac{87847.5 \times 13.44}{490.874 \times 0.94 \times 55} = +46.52 \text{ N/mm}^2 \text{ (tensile)}$$

Consider the section B - B

At section B - B, θ = 0° with respect to horizontal

$$\therefore \text{ Direct stress} \quad \sigma_d = \frac{F\cos\theta}{2A} = \frac{6500\cos o}{2 \times 490.874} = 6.621 \text{ N/mm}^2 \text{ (tensile)}$$

$$\text{Bending moment} \quad M_{b_B} = +0.182\ Fr \qquad \text{---- 25.69}$$

Where positive sign refers to tensile load and $r = r_c$

$$\therefore M_{b_B} = +0.182 \times 6500 \times 42.5 = +50277.5 \text{ Nmm}$$

This couple produces tensile stress at the inner fibre and compressive stress at the outer fibre

$$\therefore \text{ Maximum stress at the inner fibre } \sigma_{ri_B} = \sigma_d + \frac{M_{b_A} c_i}{Aer_i}$$

$$= 6.621 + \frac{50277.5 \times 11.56}{490.874 \times 0.94 \times 30} = 48.6 \text{ N/mm}^2 \text{ (tensile)}$$

$$\text{Maximum stress at the outer fibre } \sigma_{ro_B} = \sigma_d - \frac{M_{b_A} c_o}{Aer_o} = 6.621 - \frac{50277.5 \times 13.44}{490.874 \times 0.94 \times 55}$$

$$= -20 \text{ N/mm}^2 \text{ (compressive)}$$

Example : 1.24

Determine the maximum stress induced in a ring cross section of 50 mm diameter rod subjected to a compressive load of 20kN. The mean diameter of the ring is 100 mm. (V.T.U. Dec.09/Jan.10)

Solution :

The circular ring and its critical section are as shown in Fig. 1.30a and b respectively.

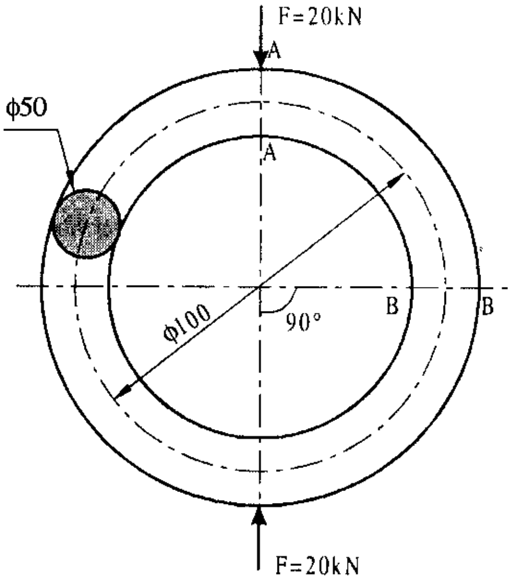


Fig. 1.30a

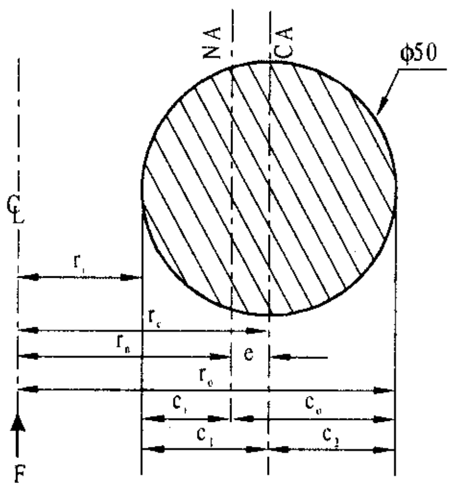


Fig. 1.30b

Inner radius $r_i = \frac{100}{2} - \frac{50}{2} = 25$ mm

Outer radius $r_o = \frac{100}{2} + \frac{50}{2} = 75$ mm

Radius of centroidal axis $r_c = \frac{100}{2} = 50$ mm

Radius of neutral axis $r_n = \frac{\left(\sqrt{r_o} + \sqrt{r_i}\right)^2}{4} = \frac{\left(\sqrt{75} + \sqrt{25}\right)^2}{4} = 46.65$ mm

$$e = r_c - r_n = 50 - 46.65 = 3.35 \text{ mm}$$

$$c_i = r_n - r_i = 46.65 - 25 = 21 - 65 \text{ mm}$$

$$c_o = r_o - r_n = 75 - 46.65 = 28.35 \text{ mm}$$

Area of cross section $= \frac{\pi}{4} \times 50^2 = 1963.5 \text{ mm}^2$

Direct stress at any cross section at an angle θ with horizontal $\sigma_d = \frac{F\cos\theta}{2A}$

Consider the section A - A

At section A - A, $\theta = 90°$ with respect to horizontal

$\therefore$ Direct stress $\sigma_d = \frac{F\cos 90}{2A} = 0$

Bending moment $M_{b_A} = +0.318\ Fr$ ---- 25.68

Where $r = r_c$, positive sign refers to compressive load

$\therefore M_{b_A} = +0.318 \times 20{,}000 \times 50 = +318000$ Nmm

This couple produces tensile stress at the inner fibre and compressive stress at the outer fibre

$\therefore$ Maximum stress at the inner fibre

$$\sigma_{ri_A} = \text{Direct stress} + \text{Bending stress}$$

$$= 0 + \frac{M_{b_A} c_i}{Aer_i} = 0 + \frac{318000 \times 21.65}{1963.5 \times 3.35 \times 25} = 41.86\ \text{N/mm}^2\ \text{(tensile)}$$

Maximum stress at the outer fibre σ_{ro_A} = Direct stress + Bending stress $= 0 - \frac{M_{b_A} c_o}{Aer_o}$

$$= -\frac{318000 \times 28.35}{1963.5 \times 3.35 \times 75} = -18.27\ \text{N/mm}^2\ \text{(compressive)}$$

Consider the section B - B

At section B - B, $\theta = 0°$ with respect to horizontal

$\therefore$ Direct stress $\sigma_d = \frac{F\cos\theta}{2A} = \frac{20{,}000 \times 1}{2 \times 1963.5} = 5.093$ N/mm² (compressive)

Bending moment $M_{b_B} = -0.182\ Fr$ ---- 25.69

Where negative sign refers compressive load and $r = r_c$

$\therefore M_{b_B} = -0.182 \times 20000 \times 50 = -182000$ Nmm

This couple produces compressive stress at the inner fibre and tensile stress at the outer fibre

$\therefore$ Maximum stress at the inner fibre

$$\sigma_{r_{i_B}} = \sigma_d - \frac{M_{b_B} c_i}{Aer_i} = -5.093 - \frac{182000 \times 21.65}{1963.5 \times 3.35 \times 25}$$

$$= -29.05\ \text{N/mm}^2\ \text{(compressive)}$$

Maximum stress at the outer fibre

$$\sigma_{r_{o_B}} = \sigma_d + \frac{M_{b_B} c_o}{Aer_o} = -5.093 + \frac{182000 \times 28.35}{1963.5 \times 3.35 \times 75}$$

$$= +5.366\ \text{N/mm}^2\ \text{(tensile)}.$$

Stress Chain Links

A link consists of two semicircles and two straight portions as shown in Fig. 1.31.

Bending moment at the point of application of load

$$M_{b_A} = \frac{Fr(2r + l)}{2(\pi r + l)} \quad \text{---- 25.76}$$

Bending moment at the section 90° away from the point of application load

$$M_{b_B} = \frac{Fr(2r - \pi r)}{2(\pi r + l)} \quad \text{---- 25.77}$$

Where $r = r_c$

The analysis of the link is similar to the analysis of closed ring.

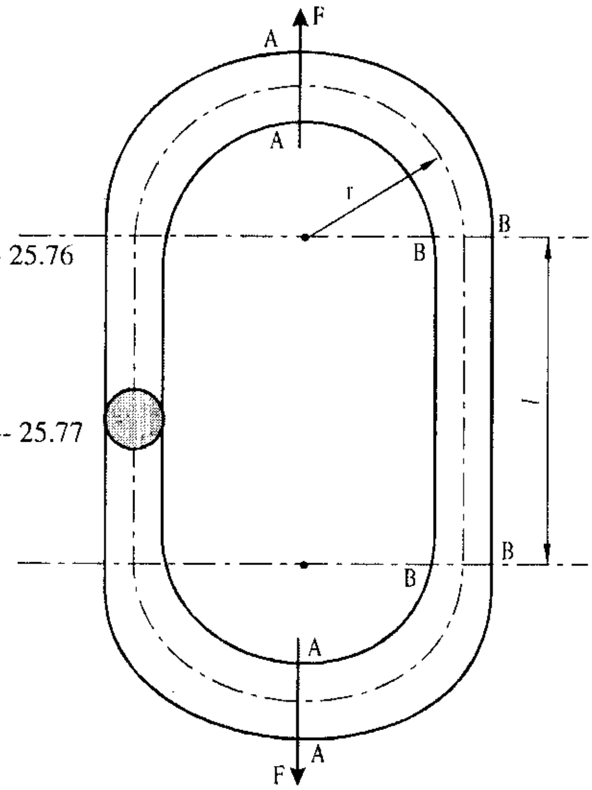


Fig. 1.31 [25.14 DDHB]

Example : 1.25

A chain link made of 40 mm diameter rod is semi circular at each end, the mean diameter of which is 80 mm. The straight sides of the link length are also equal to 80 mm. If the link carries a load of 90 kN, estimate the tensile and compressive stresses in the link along the section of load line. Also find the stresses at a section 90° away from the load line

(VTU. July/Aug. 2003)

Solution :

Refer Fig. 1.31.

l = 80 mm; d_c = 80 mm ∴ r_c = 40 mm ;

F = 90 kN = 90,000 N

Draw the critical section as shown in Fig. 1.32

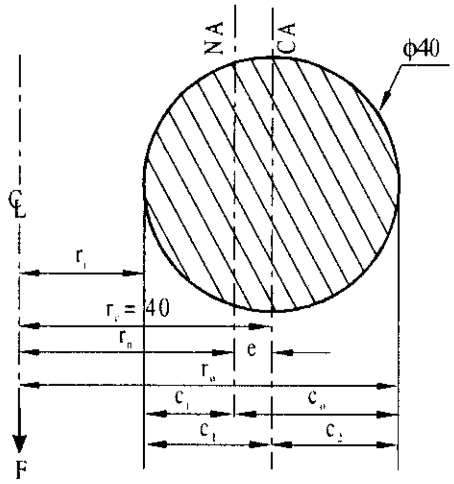


Fig. 1.32

Inner radius $r_i = 40-\frac{40}{2} = 20\,\text{mm}$

Outer radius $r_o = 40+\frac{40}{2} = 60\,\text{mm}$

Radius of centroidal axis $r_c = 40\,\text{mm}$

Radius of neutral axis $r_n = \frac{\left(\sqrt{r_o}+\sqrt{r_i}\right)^2}{4} = \frac{\left(\sqrt{60}+\sqrt{20}\right)^2}{4} = 37.32\,\text{mm}$

$$e = r_c - r_n = 40-37.32 = 2.68\,\text{mm}$$

$$c_i = r_n - r_i = 37.32-20 = 17.32\,\text{mm}$$

$$c_o = r_o - r_n = 60-37.32 = 22.68\,\text{mm}$$

Direct stress at any cross section at an angle θ with horizontal

$$\sigma_d = \frac{F\cos\theta}{2A}$$

Consider the section A - A [i.e., Along load line]

At section A - A, θ = 90° with respect to horizontal

∴ Direct stress $\sigma_d = \frac{F\cos 90}{2A} = 0$

Bending moment $M_{b_A} = \frac{Fr(2r+l)}{2(\pi r+l)}$ Where $r = r_c$ ----25.76

$$\therefore M_{b_A} = \frac{90{,}000\times 40[2\times 40+80]}{2[\pi\times 40+80]} = 1.4\times 10^6\,\text{Nmm}$$

This couple produces compressive stress at the inner fibre and tensile stress at the outer fibre

∴ Maximum stress at the inner fibre

$$\sigma_{r_{i_A}} = \sigma_d - \frac{M_{b_A}c_i}{Aer_i}$$

$$= 0 - \frac{1.4\times 10^6\times 17.32}{\left(\frac{\pi}{4}\times 40^2\right)(2.68)(20)} = -360\,\text{N/mm}^2 \text{ (Compressive)}$$

Maximum stress at the outer fibre

$$\sigma_{r_{o_A}} = \sigma_d + \frac{M_b c_o}{Aer_o}$$

$$= 0 + \frac{1.4\times 10^6\times 22.68}{\left(\frac{\pi}{4}\times 40^2\right)(2.68)(60)} = 157.14\,\text{N/mm}^2 \text{ (tensile)}$$

Consider the section B - B [i.e., 90° away from the load line]

At section B - B, $\theta = 0$ with respect to horizontal

$$\therefore \text{ Direct stress} \quad \sigma_d = \frac{F\cos 2\theta}{2A} = \frac{90{,}000\cos 0}{2\times\frac{\pi}{4}\times 40^2} = 35.81 \text{ N/mm}^2 \text{ (tensile)}$$

$$\text{Bending moment } M_{b_B} = \frac{Fr(2r-\pi r)}{2(\pi r + l)} \quad \text{Where } r = r_c \qquad \text{----25.77}$$

$$\therefore M_{b_B} = \frac{90000\times 40[2\times 40 - \pi\times 40]}{2[\pi\times 40 + 80]} = -399655.7 \text{ Nmm}$$

This couple produces tensile stress at the inner fibre and compressive stress at the outer fibre

$\therefore$ Maximum stress at the inner fibre

$$\sigma_{r_{i_B}} = \text{Direct stress} + \text{Bending stress} = \sigma_d + \frac{M_{b_B} c_i}{Aer_i}$$

$$= 35.81 + \frac{399655.7\times 17.32}{\left(\frac{\pi}{4}\times 40^2\right)(2.68)(20)} = 138.578 \text{ N/mm}^2 \text{ (tensile)}$$

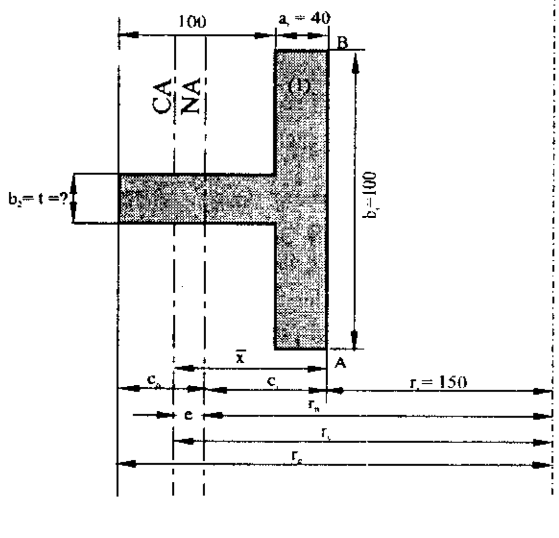
Maximum stress at the outer fibre

$$\sigma_{r_{o_B}} = \sigma_d - \frac{M_{b_B} c_o}{Aer_o} = 35.81 - \frac{399655.7\times 22.68}{\left(\frac{\pi}{4}\times 40^2\right)(2.68)(60)} = -9.047 \text{ N/mm}^2 \text{ (compressive).}$$

$\therefore$ Maximum tensile stress occurs at the outer fibre of section A - A and maximum compressive stress occurs at the inner fibre of section A - A.

Example : 1.26

Determine the value of 't' in the cross section of a curved beam shown in Fig. 1.33 such that the normal stresses due to bending at the extreme fibres are numerically equal. (VTU. Jan/Feb. 2005)



Inner radius r_i = 150mm

Outer radius r_o = 150 + 40 + 100 = 290mm

Fig. 1.33

Solution :

From Fig. 1.33, $c_i + c_o = 40 + 100 = 140\text{mm}$ ----(i)

Since the normal stresses due to bending at the extreme fibre are numerically equal

$$\frac{M_b c_i}{Aer_i} = \frac{M_b c_o}{Aer_o}$$

$$\text{i.e., } c_i = \frac{r_i}{r_o} c_o = \frac{150}{290} c_o = 0.51724\, c_o \qquad \text{----(ii)}$$

Substituting (ii) in (i)

$$0.51724\, c_o + c_o = 140$$

$$\therefore\ c_o = 92.273 \text{ mm and } c_i = 47.727 \text{ mm}$$

$$\text{Area of cross-section } A = 100 \times 40 + 100 \times t = 4000 + 100\,t$$

Distance of inner radius from neutral axis $c_i = r_n - r_i$

$$\text{i.e., } 47.727 = r_n - 150$$

$$\therefore \text{ Radius of neutral axis } r_n = 197.727 \text{ mm}$$

$$\text{Also, Radius of neutral axis } r_n = \frac{A}{b_i ln\left(\frac{r_i + a_i}{r_i}\right) + b_2 ln\left(\frac{r_o - a_o}{r_i + a_i}\right) + b_o ln\left(\frac{r_o}{r_o - a_o}\right)} \qquad \text{----25.63}$$

$b_i = 100\text{mm};\ a_i = 40\text{mm};\ b_2 = t;\ a_o = 0;\ b_o = 0;\ r_i = 150\text{mm};\ r_o = 290\text{mm};$

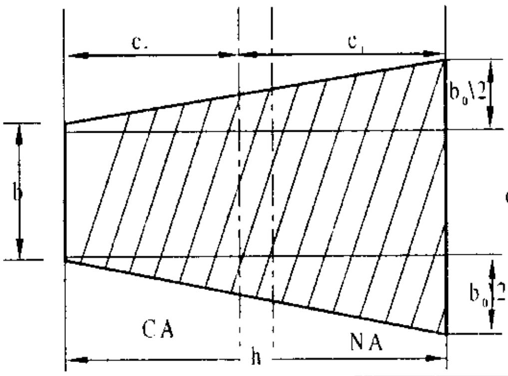
$$\therefore\ 197.727 = \frac{4000 + 100t}{100 ln\left(\frac{150+40}{150}\right) + t\, ln\left(\frac{290-0}{150+40}\right) + 0}$$

$$= \frac{4000 + 100t}{23.639 + 0.423t}$$

$$\text{i.e., } 4674.069 + 83.61t = 4000 + 100t;$$

$$\therefore\ t = 41.126 \text{mm}$$

NOTE : For trapezoidal section, From Table 2.6 (Old DDHB), Table 2.7 (New DDHB)



$$c_2 = \frac{(3b + 2b_o)h}{3(2b + b_o)}$$

REVIEW QUESTIONS

1. What are the assumptions made in finding stress distribution for a curved flexural member. Also give two differences between a straight and curved beam

   VTU, Aug. 2001, Feb. 2003

2. Discuss the stress distribution pattern in curved beams when compared to straight beam with sketches

   VTU, Feb. 2002, July/Aug. 2004

3. Derive an expression for stress distribution due to bending moment in a curved beam

   BU Sep/Oct 2000, Jan/Feb. 2004

EXERCISES

1. Determine the force F that will produce a maximum tensile stress of $60 N/mm^2$ in section A - B and the corresponding stress at the section C - D (Fig. 1.34)

2. A crane hook has a section of trapezoidal. The area at the critical section is $115\left(\frac{75+25}{2}\right)$ mm^2. The hook carries a load of 10kN and the inner radius of curvature is 60 mm. Calculate the maximum tensile, compressive and shear stress.

   Hint : $b_i = 75$ mm; $b_o = 25$ mm; h = 115 mm

3. A closed ring is made of 40 mm diameter rod bent to a mean radius of 85 mm. If the pull along the diameter is 10,000 N, determine the stresses induced in the section of the ring along which it is divided into two parts by the direction of pull.

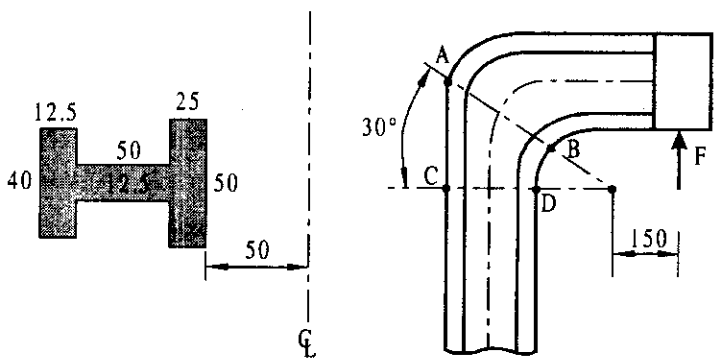


Fig. 1.34

4. Determine of value of t in the cross section of a curved beam shown in Fig.1.35 such that the normal stresses due to bending at the extreme fibers are numerically equal.

VTU, Jan/Feb. 2005

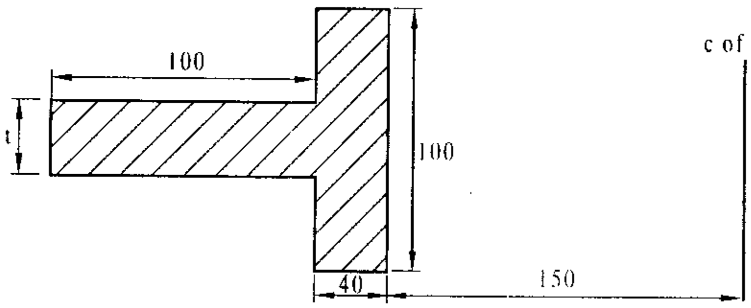


***Fig.*1.35**

5. Determine a safe value for load P for a machine element loaded as shown in Fig. 1.36 limiting the maximum normal stress induced on the cross section XX to 120 MPa.

VTU, Jan/Feb. 2006

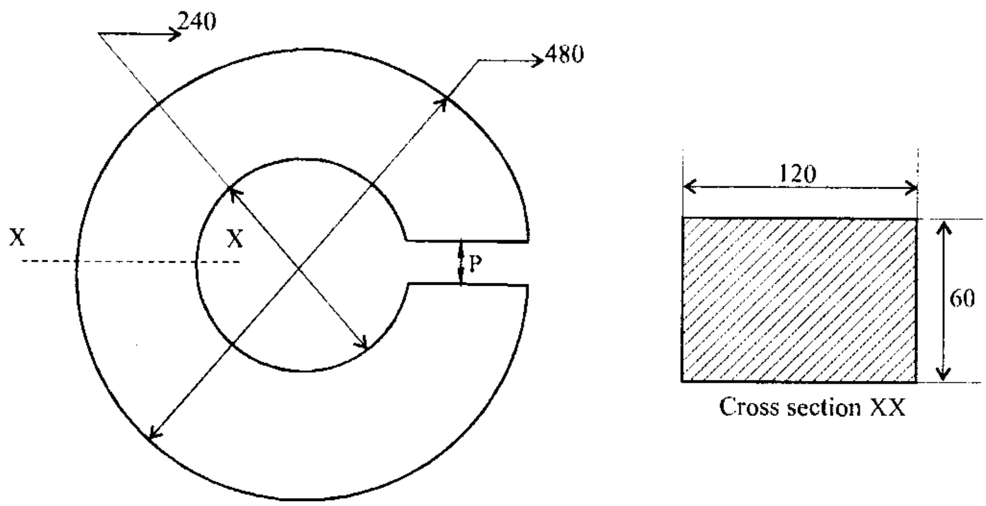


***Fig.* 1.36**

6. a. Explain why curved beams have to be analysed for stresses specially when we already have straight beam equations for determining the stresses?

   b. The beam shown in Fig. 1.37 is subjected to a load of 50 kN. Determine the stresses at the inner and outer fibers. Plot the stress distribution. **VTU, July 2006**

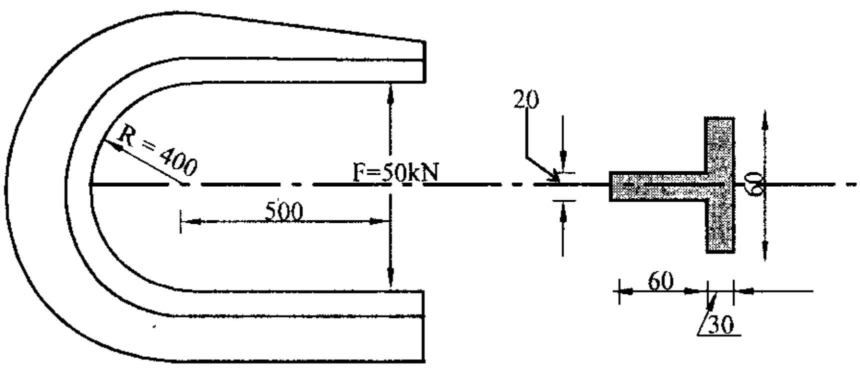


Fig. 1.37

7. Compute the combined stresses at the inner and outer fibres in the critical cross section of a crane hook which is required to lift loads up to 25 kN. The hook has trapezoidal cross section with parallel sides 60 mm and 30 mm, the distance between them being 90 mm. The inner radius of the hook is 100 mm. The load line is nearer to the inner surface of the hook by 25 mm than the centre of curvature at the critical section. What will be the stresses at the inner and outer fibre, if the beam is treated as straight beam for the given load. **VTU, Dec 2006 Jan. 2007**

8. Calculate the stresses at points A and B for a circular beam as shown in Fig.1.38. The circular beam is subjected to a compressive load of 6 kN. **VTU, July 2007**

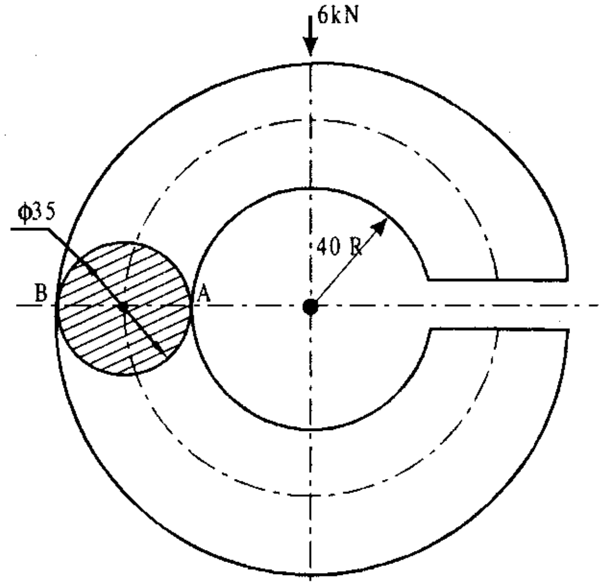


Fig.1.38

9. The section of a crane hook is trapezoidal, whose inner and outer sides are 90 mm and 25 mm respectively and has a depth of 116 mm. The center of curvature of the section is at a distance of 65 mm from the inner side of the section and load line passes through the center of curvature. Find the maximum load the hook can carry, if the maximum stress is not to exceed 70 MPa. **VTU, Dec. 07/Jan. 08**

10. a) Differentiate between a straight beam and a curved beam with stress distribution in each of the beam.

    b) Fig. 1.39 shows a 100 kN crane hook with a trapezoidal section. Determine stress in the outer, inner, Cg and also at the neutral fibre and draw the stress distribution across the section AB. **VTU, Jun/July. 08**

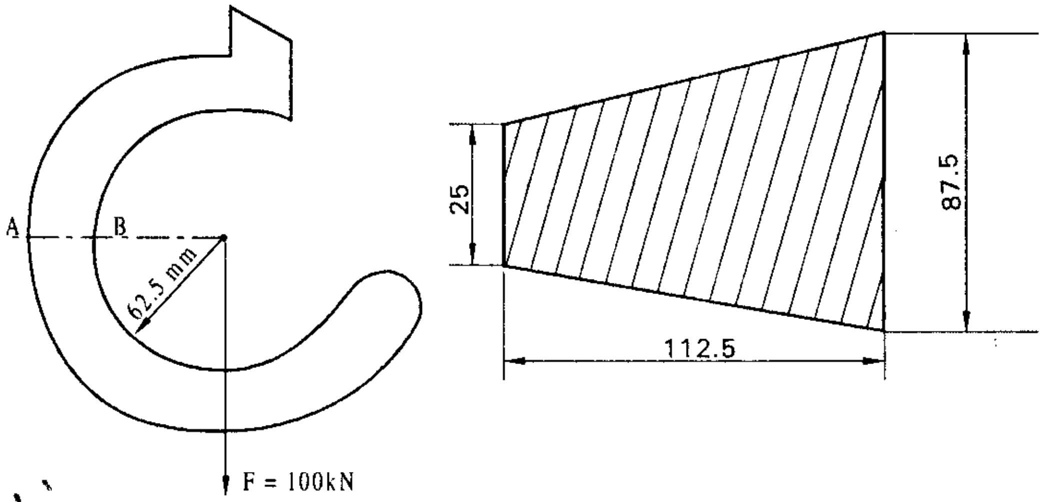


Fig.1.39

11. A closed ring is made up of 50 mm diameter steel bar having allowable tensile stress of 200 MPa. The inner diameter of the ring is 100 mm. For load of 30 kN find the maximum stress in the bar and specify the location. If the ring is cut as shown in part -B of Fig. 1.40, check whether it is safe to support the applied load. **VTU, Dec. 08/Jan. 09**

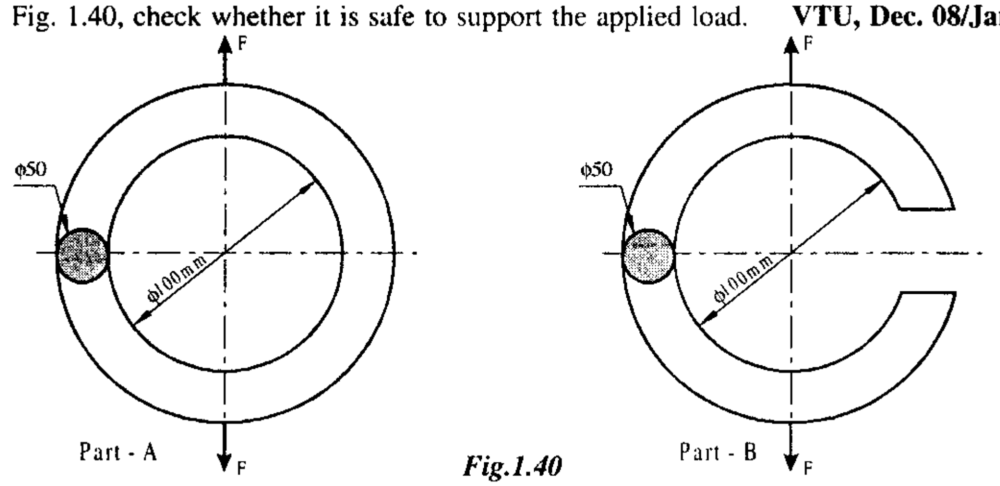


Fig.1.40

UNIT 2

CYLINDERS AND CYLINDER HEADS

2.1 INTRODUCTION

Cylindrical pressure vessels are classified into two groups. (i) Thin cylinders and (ii) Thick cylinders. A cylinder is considered to be thin when the ratio of its wall thickness to the internal radius is less than $\frac{1}{10}$. In thin cylinder the stress distribution is assumed to be uniform over the thickness of wall.

2.2 STRESSES IN A THIN CYLINDER

When a thin cylinder is subjected to internal pressure, its walls are subjected to two types of tensile stresses.

(i) Circumferential stress or Hoop stress or Tangential stress

(ii) Longitudinal stress

The stress acting along the circumference of the cylinder is called circumferential stress and the stress acting along the length of the cylinder is called longitudinal stress. Circumferential stress is also called as hoop stress. The stress set up in two troughs is circumferential stress and the stress set up in two cylinders is longitudinal stress.

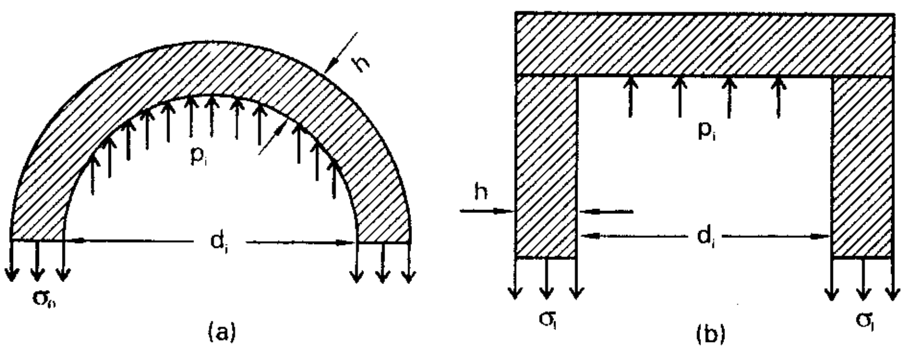


Fig. 2.1